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Superluminal Motion of Free Spin-half Particles in a Fiber Bundle Formalism

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Authors' contributions

The corresponding author EDKG solely designed, analyzed, interpreted, and prepared this manuscript as well as its underlying article [1]. They form part of his Doctoral Thesis which was carried out under the supervision of authors FTO and EP. All authors read and approved the final manuscript.

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Review Article

ABSTRACT

The hypothesis of the spin of an electron, leading to its angular momentum, is no longer an open question, in science. Experimental evidence like the hydrogen fine structure and the Stern-Gerlach experiment in the 1920s and, recently, Nuclear Magnetic Resonance (NMR) have for long paved the way to definitely end this debate. Equipped with this mathematical machinery potent enough to handle the theories which are of interest to us, the expectation value of the overall relative linear velocity component of a fermion field was investigated in a previous paper, found quantized, and exceeding the speed of light. In the present article, we aim to review and describe this result in the framework of a fiber bundle theory. Using a method developed in the 1980s by Zimmer for Dynamical-System theory, we explicate the feasibility of the superluminal free electron and neutrino result in bundle language.

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1 INTRODUCTION

The very particle-concept of an electron and its mechanical picture as a rigid body rotating about its axis constitute a foundation of the hypothesis of the spin of the electron, as proposed by Kronig, Uhlenbeck, and Goudsmit [2] [3], though this picture had faced a rejection, at its conception, on the collective advice of Pauli, Kramers, and Heisenberg, [4]. Of course, the speed of rotation v, calculated from the spin angular momentum magnitude of $\hbar/2$ and the classical electron radius of $r = 2.81794 \times 10^{-15} metres$, that is:

$$v = r\omega = \frac{5\hbar}{4mr} = \frac{5 \times 1.05457 \times 10^{-34} Js}{4\left(9.10938 \times 10^{-31} kg\right)\left(2.81794 \times 10^{-15} metres\right)} = 513.5 \times 10^{8} m/s$$

is in excess (more than 100 times) of the speed of light, where m denotes the electron mass and ω its angular frequency.

In fact, from the beginning of the concept of spin to date, there continue to exist conflicting opinions among scientists on the subject of faster-than-light phenomenon, divided into advocate (cf.[5]) and those who deny this possibility [6] [7]. At present (from 2008 to date), scientists are yet to finally decide, in respect of the conflicting conclusions of CERN experiments, on superluminal neutrino result.

Up until 1900, it was quite impossible in Maxwell and Kelvin synthesis to attribute a measure of mass to an electromagnetic radiation and, let alone, to imagine an equivalence between mass and energy, as it is today for current level of understanding of science to accept free spin-1/2 particles superluminal motion possibility.

Here, we explain why causality violation, due to superluminal motion, should not be considered as a paradoxical occurrence and signify the dead end of scientific unfolding. Since spacetime is curved, each "local" system of rectangular coordinate axes could be thought of as an orthogonal intersection of two *great circles* (longitude and latitude) [8], at the infinity scale. The linear motion, in spacetime, of a superluminal particle departing from the origin along any of these great circles (longitudinal, for example) will be circular from one hemisphere to the other; so that when the particle (under 'excessive' speed) is found in the other hemisphere, its (temporal) component will appear negative on the "local" picture of the coordinate axes. This is one way we may comprehend the change to negative temporal component of variation in spin angular momentum, leading to "apparent" backward in time motion in the light cone (see Fig. 1), and as portrayed in [1].

Thus, in reality, there is no backward in time scenario, and the resulting causality violation may be regarded as just a mirage.

In our previous article [1], with title: *Investigation of Superluminal Motion of Free Spin-half Particles in Spacetime*, we utilized the symmetrized Dirac Lagrange density to derive two important results. These are the variations in mixed space-time components of orbital angular momentum L_{03}, and intrinsic spin angular momentum S_{03}, given by:

$$\delta L_{03}\left[\psi(x), \psi^{\dagger}(x)\right] = S_{03}\left[\psi(x), \psi^{\dagger}(x)\right] = \frac{1}{2}\int d^{3}x\psi^{\dagger}(x)\,\sigma_{03}\psi(x) \qquad (1.1)$$

and

$$\delta S_{03}\left[\psi(x), \psi^{\dagger}(x)\right] = -S_{03}\left[\psi(x), \psi^{\dagger}(x)\right] = -\frac{1}{2}\int d^{3}x\psi^{\dagger}(x)\,\sigma_{03}\psi(x), \qquad (1.2)$$

respectively, where $\psi(x,t)$ denotes the Dirac wave function. If we insert the solutions of the Dirac equation [9] [10] [11], with the free wave propagating in the $z-$direction with positive energy

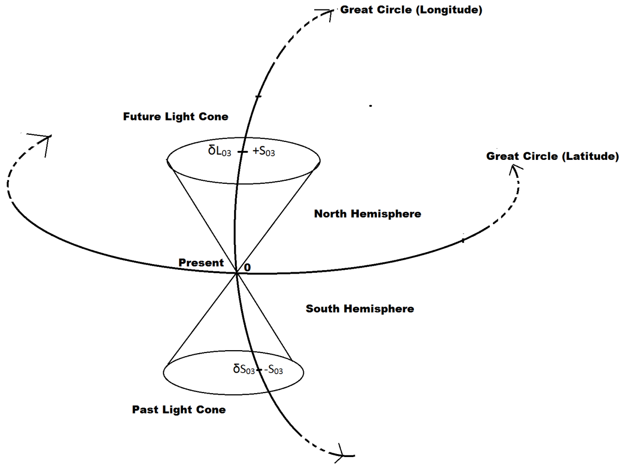

Fig. 1. Representation of the light cone in a coordinate system of two rectangular great circles (longitude and latitude) of curved spacetime at infinity scale

$$\Psi_{p,\lambda,+1/2} = N \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \frac{c\hat{\sigma}_z p}{m_0 c^2 + E_p} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} \exp\left[i\left(pz - E_p t\right)/\hbar\right], \tag{1.3}$$

(1.1) and (1.2) lead to the following equations for variations in orbital and spin angular momenta [1]:

$$\begin{aligned} \delta L_{03}(x) &= \frac{1}{2}\int d^3x N \begin{pmatrix} 1 & 0 & \frac{c\hat{\sigma}_z p}{m_0 c^2 + E_p} & 0 \end{pmatrix} \exp\left[-i\left(pz - E_p t\right)/\hbar\right] \\ & \quad \times \sigma_{03} N \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \frac{c\hat{\sigma}_z p}{m_0 c^2 + E_p} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} \exp\left[i\left(pz - E_p t\right)/\hbar\right] \\ &= \frac{1}{2}\sigma_{03} x_\mu = \frac{1}{2}\sigma_{03} \begin{pmatrix} t \\ z \\ y \\ x \end{pmatrix} \end{aligned} \tag{1.4}$$

and

$$\delta S_{03}(x) = -\frac{1}{2}\sigma_{03} x_\mu = \frac{1}{2}\sigma_{03}\left(-x_\mu\right) = \frac{1}{2}\sigma_{03} \begin{pmatrix} -t \\ -z \\ -y \\ -x \end{pmatrix}. \tag{1.5}$$

Equation (1.5) violates causality, while (1.4) does not. The matrix of the generalized angular momentum transformation of the field is obtained as

$$\begin{aligned} r(0) &= \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{4} \\ 0 & 0 & -\frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & 0 & -\cos(V/c) \\ 0 & 0 & -\cos(V/c) & 0 \\ 0 & \cos(V/c) & 0 & 0 \\ \cos(V/c) & 0 & 0 & 0 \end{pmatrix}, \end{aligned} \tag{1.6}$$

this culminates into the superluminal motion of the free spin-1/2 field, of overall relative linear, and quantized, velocity of expectation value (cf.[1]):

$$V \simeq \left(\frac{21\pi}{50} + 2k\pi \right) \times c, \quad (k = 0, 1, 2, ...)\,, \tag{1.7}$$

where c denotes the speed of light, as it should be in accordance with quantum mechanics prescription that discretized restraints should account for discontinuous energy levels and spectra.

Observe that this superluminal result is not without a familiar precedent. Photon velocity in *Multi-level Universes World* is also quantized, and the limiting speed of a particle with zero mass (i.e., of mass $m = 0$) could be superluminal of velocity

$$C_S = 1c,\ 2c,\ 3c,\ 4c, ...\ or \quad 1c,\ 3c,\ 5c,\ 7c, ...,$$

For more information on this, see [12] [13] [14] [15] [16] [17]. The matrix $r\,(0)$ in (1.6) is a non-Hermitian, antisymmetric, rotation operator. In our attempt to identify the family of which this matrix originates, it is of interest to note that the infinite dimensional spin angular momentum operators of spin 1 boson particles, of the Infinite Dimensional Lorentz Group are given by

$$\mathbb{k} = \begin{bmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 0 & 0 & 0 & 0 & & & & & & & & 5i & \cdot \\
\cdot & 0 & 0 & 0 & 0 & & & & & & & 4i & & \cdot \\
\cdot & 0 & 0 & 0 & 0 & & & & & & 3i & & & \cdot \\
\cdot & 0 & 0 & 0 & 0 & & & & & 2i & & & & \cdot \\
\cdot & & & & & 0 & 0 & 0 & i & & & & & \cdot \\
\cdot & & & & & 0 & 0 & 0 & 0 & & & & & \cdot \\
\cdot & & & & & 0 & 0 & 0 & 0 & & & & & \cdot \\
\cdot & & & & & i & 0 & 0 & 0 & & & & & \cdot \\
\cdot & & & & 2i & & & & & 0 & 0 & 0 & 0 & \cdot \\
\cdot & & & 3i & & & & & & 0 & 0 & 0 & 0 & \cdot \\
\cdot & & 4i & & & & & & & 0 & 0 & 0 & 0 & \cdot \\
\cdot & 5i & & & & & & & & 0 & 0 & 0 & 0 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{bmatrix}, \tag{1.8}$$

or

$$\mathbb{k} = \begin{bmatrix}
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & 0 & 0 & 0 & 0 & & & & & & & & 5 & \cdot \\
\cdot & 0 & 0 & 0 & 0 & & & & & & & 4 & & \cdot \\
\cdot & 0 & 0 & 0 & 0 & & & & & & 3 & & & \cdot \\
\cdot & 0 & 0 & 0 & 0 & & & & & 2 & & & & \cdot \\
\cdot & & & & & 0 & 0 & 0 & 1 & & & & & \cdot \\
\cdot & & & & & 0 & 0 & 0 & 0 & & & & & \cdot \\
\cdot & & & & & 0 & 0 & 0 & 0 & & & & & \cdot \\
\cdot & & & & & -1 & 0 & 0 & 0 & & & & & \cdot \\
\cdot & & & & -2 & & & & & 0 & 0 & 0 & 0 & \cdot \\
\cdot & & & -3 & & & & & & 0 & 0 & 0 & 0 & \cdot \\
\cdot & & -4 & & & & & & & 0 & 0 & 0 & 0 & \cdot \\
\cdot & -5 & & & & & & & & 0 & 0 & 0 & 0 & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot
\end{bmatrix}, \tag{1.9}$$

which are constructed from the six 4×4 Lorentz group operators, involving the Lorentz boost. Then, under certain proceedings (which we cannot display in this so restricted arena) from here, one can extend Einstein Special Relativity to the so-called *Worm Hole Special Relativity* in *Multi-level Universes World*, [12] [13] [14] [15] [16] [17]. And our matrix $r\,(0)$ of generalized angular momentum of the free spin-1/2 field seems somewhat to be familiar with the part of

$\mathbb{k}$ shown in (1.9); they are both non-Hermitian, antisymmetric, and second-diagonal operators, for example.

Moreover, the propagation is helical and invariant under spatial rotations, [18] [19] [20]. This invariance under spatial rotations of the spin-1/2 fields relates to the Invariance Reduction Theorem (IRT) which renders the existence and uniqueness problems of invariant fields into a problem of Topological Dynamics involving techniques from Ergodic Theory and Homotopy Theory, [21]. It is our goal to enlarge the audience of faster-than-light free spin-1/2 field by attempting its interpretation in another mathematical language, that of fiber bundle.

Section 2 of this paper reviews a method developed in the 1980s by Zimmer for Dynamical-System theory [22]. By this, the notions of particle-spin motion and field motion can be generalized. We elect to employ a discrete-time formalism, though a continuous-time treatment is also feasible. Two major theorems are presented, the Decomposition Theorem, which allows one to compare different invariant fields and the Invariant Reduction Theorem, which gives new insights into the existence and uniqueness problems of invariant fields (and in particular invariant spin fields). It turns out that the well established notions of invariant polarization field and invariant spin field are generalized to invariant (E, l)-fields. Here the notation (E, l) will mean that E is a topological space and that the function $l : SO(3) \times E \rightarrow E$ is a continuous $SO(3)$ −action, i.e., $l(I; x) = x$ and $l(r_1 r_2; x) = l(r_1; l(r_2; x))$, where r_i $(i = 1, 2)$ are rotation matrices. With the flexibility in the choice of (E, l) we have a unified way to study the dynamics of spin-$1/2$ particles. Accordingly the special cases $(E, l) = (\mathbb{R}^3, l_{1/2})$ is discussed in some detail. Section 3 looks at interpreting the result of superluminal free spin-1/2 field in the fiber bundle formalism.

2 THE FORMALISM

Bundle aspects are the origin of our formalism (see [23]) and therefore supply a steady flow of ideas, many of which are not even mentioned here (e.g., algebraic hull, characteristic class, rigidity). We begin this section with a formal definition of fiber bundle, as distilled from [24] [25] [26], together with an underlying short bundle theory.

Definition *A fiber bundle is a structure (E, B, π, F), where E, B, and F are topological spaces and $\pi : E \rightarrow B$ is a continuous surjection satisfying a local triviality condition outlined below. The space B is called the base space of the bundle, E the total space, and F the fiber. The map π is called the projection map (or bundle projection). We shall assume, in the local triviality condition which follows, that the base space B is connected. It is required that for every x in E, there is an open neighborhood $U \subset B$ of $\pi(x)$ (which will be called a trivializing neighborhood) such that there is a homeomorphism φ: $\pi^{-1}(U) \rightarrow U \times F$ (where $U \times F$ is the product space) in such a way that π agrees with the projection onto the first factor. That is, the following diagram (Fig. 2*) should commute: where $proj_1 : U \times F \rightarrow U$ is the natural projection and φ: $\pi^{-1}(U) \rightarrow U \times F$ is a homeomorphism. The set $\{(U_i, \varphi_i)\}$ of all (U_i, φ_i) is called a *local trivialization* of the bundle.

Thus for any p in B, the preimage $\pi^{-1}(\{p\})$ is homeomorphic to F (since $proj_1(\{p\})$ clearly is) and is called the fiber over p. Every fiber bundle $\pi : E \rightarrow B$ is an open map, since projections of products are open maps. Therefore B carries the quotient topology determined by the map π.

A fiber bundle (E, B, π, F) is often denoted $F \longrightarrow E \stackrel{\pi}{\longrightarrow} B$ which, in analogy with a short exact sequence, indicates which space is the *fiber*, *total space* and *base space*, as well as the map from total to base space.

A smooth fiber bundle is a fiber bundle in the category of smooth manifolds. That is, E, B, and F are required to be smooth manifolds and all the functions above are required to be smooth maps.

Fiber bundles are the natural language for the description of a gauge theory. Locally a *principal bundle* $\mathbf{P}$ is the product of a *structure group* (i,e., gauge group) with the base space (space-time), but globally twists can appear given by the translation functions, [27]. For example, a cylinder is the trivial product of a line with a circle. A Mӧbius strip would be a nontrivial bundle with one 180^o twist as the line went aroud the circle. Locally a Mӧbius strip and a circle are identical but globally quite different.

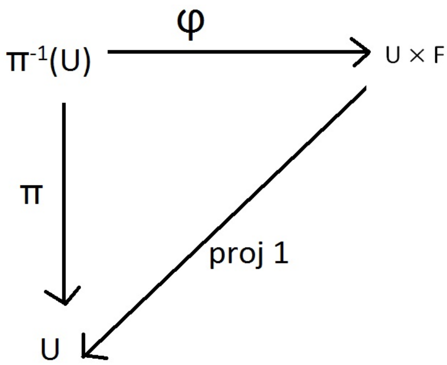

Fig. 2. Commutation diagram

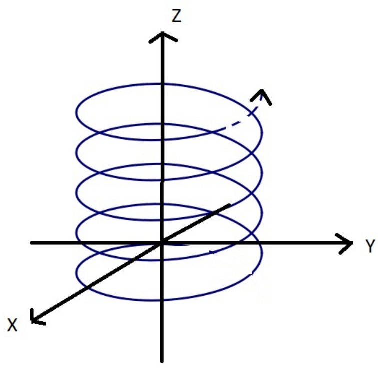

Fig. 3. Invariant helical motion of free spin-half fields: Concentric circles centered at the origin; invariant polarization describing the spin equilibrium of a bunch

2.1 Homotopy in the Rotation and Lorentz Groups

The description of intrinsic spin, whether for bosons or for fermions, is in terms of fiber bundles with an $SO(3)$ structure group. It is clearly of some interest then to understand the "loop structure", i.e., the fundamental group, of $\mathbb{R}^3 \times SO(3)$. Notice that $SO(3)$ does indeed have a natural topology. The entries of a 3×3 matrix can be strung out into a column matrix which can be viewed as a point in $\mathbb{R}^9$. Thus, $SO(3)$ can be viewed as a subset of $\mathbb{R}^9$ and therefore inherits a topology as a subspace of $\mathbb{R}^9$, [28]. A considerably more informative "picture" of $SO(3)$ can be obtained as follows: Every rotation of $\mathbb{R}^3$ can be uniquely specified by an axis of rotation, an angle and a sense of rotation about the axis. We claim that all of this information can be codified in a single object, namely, a vector in $\mathbb{R}^3$ of magnitude at most π. Then the axis of rotation is the line along $\vec{n}$, the angle of rotation is $|\vec{n}|$ and the sense is determined by the "right hand rule". Notice that a rotation along $\vec{n}$ through an angle θ with $\pi \leq \theta \leq 2\pi$ is equivalent to a rotation along $-\vec{n}$ through $2\pi - \theta$ so the restriction on $|\vec{n}|$ is necessary (although not quite sufficient) to ensure that the correspondence between rotations and vectors be one-to-one. The set of vectors $\vec{n}$ in $\mathbb{R}^3$ with $|\vec{n}| \leq \pi$ is just the closed ball of radius π about the origin. However, a rotation about $\vec{n}$ through π is the same as a rotation about $-\vec{n}$ through π so *antipodal points* on the boundary of this ball represent the same rotation and therefore must be identified in order that this correspondence with rotations be bijective. Carrying out this identification yields real projective $3-$space (topologically, the radius of the ball is irrelevant, of course).

Having found the structure group of our fiber bundle, we now look for the appropriate base space. Consider the simple case of static particles with spin. A very natural choice for the base space is thus $\mathbb{R}^3$ minus the point at the origin where the particle is assumed to be. Thus we take our base space to be $\mathbb{R}^3 - \{0\}$. Note that Minkowski space minus the world line of a particle is contractible to $\mathbb{R}^3 - \{0\}$ which, in turn, can be retracted to S^2 sphere without changing the fiber bundle. Thus we are led to a principal fiber bundle, $\mathbf{P}$, with structure group $SO\,(3)$ and base space the two-sphere, S^2, in connection with describing static particles with spin. The present work has nothing to do with static spinning fermions, of course.

In our present case of moving spin-1/2 particles, we consider the torus T^d as the locus of the position z of our particle with spin-1/2. It is known that $T^d = S^d \times S^d$, where S^d is the $d-$sphere and, generally $d = 1, 2, 3$. The "unreduced" principal bundle underlying our formalism is a product principal bundle ($T^d \times SO(3)$,p_d,T^d,R_d) with bundle space $T^d \times SO(3)$, base space T^d, bundle projection $p_d(z, r){:=}z$ (with z in T^d and $r{\in}SO(3)$), and structure group $SO(3)$. So R_d $:SO(3) \times T^d \times SO(3) {\to} T^d \times SO(3)$ is an $SO(3)-$action defined by $R_d(r; z^{'}, r^{'}){:=}$ ($z^{'}$, $r^{'} r^t$), where the upper index t means "transpose". The reductions are just the principal subbundles of the unreduced bundle. So they are uniquely determined by their bundle space X which of course is a subgroup of $T^d \times SO(3)$.

2.2 Particle-spin Motion

For given (E, l) each particle carries, in addition to its position z on the torus T^d an $E-$valued quantity x we call spin. The one-turn particle-spin map is the function $P[j, A] : T^d {\times} E \to T^d {\times} E$, defined by

$$P[j, A]\,(z, x) = (j\,(z)\,, l\,(A\,(z)\,, x))\,, \tag{2.1}$$

where $j \in Homeo\left(T^d\right)$ is the one-turn particle map (e.g., linear translation on the torus) and $A \in C\left(T^d, SO\,(3)\right)$ is the one-turn spin transfer matrix. Here $Homeo(T^d)$ denotes the set of homeomorphisms on T^d, $C\,(X, Y)$ denotes the set of continuous functions from X to Y (for the spinor formalism our formalism is obtained by simply replacing $SO\,(3)$ by $SU\,(2)$. In the present formalism, (2.1) is the most general description of particle-spin dynamics and the choice of (E, l) depends on the situation, e.g., $(E, l) = \left(\mathbb{R}^3, l_{1/2}\right)$ for spin-$1/2$ particles. We work in the framework of topological dynamical systems and therefore A, j, l are continuous functions. This condition could be strengthened to A, j, l being smooth functions.

2.3 Field Motion and Invariant Fields

We are primarily interested in the field dynamics induced by the particle-spin dynamics. Let f : $T^d \to E$ be an $E-$valued field on T^d and set $x = f\,(z)$ in (2.1). Then after one turn z becomes $j\,(z)$ and the field value at $j\,(z)$ becomes $l\,(A\,(z)\,; f\,(z))$. Observe that not after one turn as in [21], but rather after two turns (to generalize the case to spin-1/2 particles, as well) the field f becomes the field $f^{'} : T^d \to E$ where $f^{'}\,(z) := l\left(A\left(j^{-1}\,(z)\,; f\left(j^{-1}\,(z)\right)\right)\right)$. Thus we have the field map

$$f \mapsto f^{'} = l\left(A \circ j^{-1}; f \circ j^{-1}\right), \tag{2.2}$$

where $\circ$ denotes the composition of functions. We call $f \in C\left(T^d, E\right)$ an "invariant (E, l)-field of (j, A)" if it is mapped by (2.2) into itself, i.e., if

$$f \circ j = l\,(A, f)\,. \tag{2.3}$$

We call (2.3) the *(E, l)-stationarity equation of (j, A)*. Our main focus is on the existence of solutions of (2.3) as this is what describes the spin equilibrium of a bunch. In the important case where $(E, l) = \left(\mathbb{R}^3, l_{1/2}\right)$, an invariant $(E, l)-$field f such that $\|f\| = 1$ is called an *invariant spin field* (ISF). This completes our introduction to the formalism.

2.4 The Set $\Sigma_x[f]$ and Its Invariance

Let $E_x := \{l\,(r; x) : r \in SO\,(3)\}$; so, clearly, $E_x{=}\{S{\in}\mathbb{R}^3 : \|S\|{=}\|x\|\}$ is a sphere centered at (0,0,0). Then the E_x partition E and each set $T^d {\times} E_x$ is invariant under the particle-spin motion of (2.1) and so we have "decomposed" $T^d \times E$. Let

$$\Sigma_x[f] := \left\{ z \in T^d : f(z) \in E_x \right\}. \qquad (2.4)$$

The nonempty sets among the $\Sigma_x[f]$ form a partition of T^d and tell us how the values of f are distributed, i.e., $z \in \Sigma_x[f]$ iff $f(z) \in E_x$. It follows from the definition of $\Sigma_x[f]$ and (2.2) that $\Sigma_x[f']=j(\Sigma_x[f])$. Thus if f is invariant then every $\Sigma_x[f]$ is invariant under j and T^d is partitioned into f -dependent invariant sets for the particle dynamics, an interesting fact in its own right.

We can now state three facts related to the existence of invariant fields. Firstly, if there exists an x such that $\Sigma_x[f]$ is not invariant then f is not an invariant field. Secondly, if $\Sigma_x[f]$ is nonempty, let $f_x \in C(\Sigma_x[f], E_x)$ where $f_x(z)= f(z)$. Then f is invariant iff $f_x(j(z)) = l(A(z); f_x(z))$ for every nonempty $\Sigma_x[f]$.

Finally, suppose that j is topologically transitive [e.g., off orbital resonance (see definition below)]. This means that a $z_0 \in T^d$ exists such that $B:=\{j^n(z_0): n=0,\pm1,\pm2,...\}$ is dense in T^d, i,e., that the closure $\bar{B}$ of B equals T^d. Let f be invariant and pick x such that $z_0 \in \Sigma_x[f]$ then $B \subset \Sigma_x[f]$. Assume E is Hausdorff (e.g., the topology is from a metric). Then it follows that $\Sigma_x[f]$ is closed and , since $\bar{B}=T^d$, we have $\Sigma_x[f] = T^d$. Thus topological transitivity and the Hausdorff property imply an invariant f takes values only in one E_x. The so-called ISF-conjecture claims, for $(E,l)=(\mathbb{R}^3, l_{1/2})$, that if j is topologically transitive, then ISF exists.

The above considerations (which are sufficient for our purpose in this paper) can however be formalized into the following two theorems [21]: the Decomposition Theorem (DT) and the Invariant Reduction Theorem (IRT); hence no need arises to elaborate on these.

2.5 The Decomposition Theorem (DT)

Let E be Hausdorff. It is natural to ask about the relation between the dynamics on two distinct invariant sets $T^d \times E_x$, $T^d \times E_y$. Consider the particle-spin trajectories defined by $(z(n+1), x(n+1))=P[j,A](z(n),x(n))$ where $(z(0),x(0))=(z_0,x_0)$ is given with $x_0 \in E_x$. In addition, suppose there exists $\beta \in C(E_x, E_y)$ such that for every particle-spin trajectory $(z(n),x(n)) \in T^d \times E_x$, the function $(z(n),\beta(x(n))) \in T^d \times E_y$ is particle-spin trajectory. A necessary and sufficient condition for β to have this property is that $\beta(l(r;\xi))=l(r;\beta(\xi))$ for all $r \in SO(3)$, $\xi \in E_x$ and this is true iff $r_0 \in SO(3)$ exists such that $\{r_0 r r_0^t : r \in H_x\} \subset H_y$. Here, the subgroup H_η of $SO(3)$ is defined by $H_\eta := \{r \in SO(3) : l(r;\eta)=\eta\}$ for every $\eta \in E$. The proof of this is constructive showing that β can be defined by $\beta(l(r;x)) := l(r r_0^t; y)$. Furthermore, it can be shown that if f is an invariant (E,l) −field of (j,A) which takes values only in E_x, then $g \in C(T^d, E)$, defined by $g(z) := \beta(f(z))$, is an invariant field taking values only in E_y. In summary, the DT classifies invariant fields in terms of the functions β, i.e., in terms of the subgroups H_x of $SO(3)$.

2.6 The Invariant Reduction Theorem (IRT)

Let $f \in C(T^d,E)$, $x \in E$, $\check{\Sigma}_x[f] := \{(z, r) \in (T^d \times SO(3)) : l(r,x)=f(z)\}$ and $\check{P}[j,A] \in Homeo(T^d \times SO(3))$ with $\check{P}[j,A](z, r) := (j(z), A(z)r)$. Then the IRT states that f satisfies (2.3) iff $\check{\Sigma}_x[f]$ is invariant under $\check{P}[j,A]$ for every $x \in E$. For brevity sake, it should be noted that, as its name suggests, the IRT identifies the dynamical invariance of the reductions with the invariance of the labeling fields f. It has a supplement called the Cross Section Theorem (CST) which gives valuable information to be used when applying the IRT to (2.3), and the closely related Normal Form Theorem (NFT) ties invariant fields with the notion of normal form.

3 BUNDLE INTERPRETATION OF THE SPECIAL CASE OF SUPERLUMINAL FREE SPIN-1/2 PARTICLES IN SPACETIME

According to proceedings from above, for spin-1/2 particles in 4-dimensional spacetime, the characteristic condition and the most important

(E, l) is given by

$$E = \mathbb{R}^{1,3} \quad and \quad l_{1/2}\left(r; S\right) := rS. \qquad (3.1)$$

Here r is a 4×4 rotation matrix and $S \in \mathbb{R}^{1,3}$ is a 4-vector of the Lorentzian space $\mathbb{R}^{1,3}$. Clearly $E = \mathbb{R}^{1,3}$ is Hausdorff (since $\mathbb{R}^3$ is) and the E_x are concentric spheres centered at $(0, 0, 0, 0)$ (see Fig. 3) and the field map (2.2) gives $f^{'}(z) {=} A\left(j^{-1}(z)\right) f\left(j^{-1}(z)\right).$

The invariant $\left(\mathbb{R}^{1,3}, l_{1/2}\right)-$fields f are just the invariant polarization fields describing the spin equilibrium of a bunch and, calculating the norm of the field (using (1.3)), we have

$$\begin{aligned}
\parallel f \parallel^2 &= \Psi^{\dagger}_{p,\lambda,+1/2} \cdot \Psi_{p,\lambda,+1/2} \\
&= N \begin{pmatrix} 1 & 0 & \frac{c\hat{\sigma}_z p}{m_0 c^2 + E_p} & 0 \end{pmatrix} \exp\left[-i\left(pz - E_p t\right)/\hbar\right] \\
&\quad \times N \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \frac{c\hat{\sigma}_z p}{m_0 c^2 + E_p} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} \exp\left[i\left(pz - E_p t\right)/\hbar\right] \\
&= 1 = \parallel f \parallel; \qquad (3.2)
\end{aligned}$$

and this shows they are just the invariant spin fields.

Next, application of the formalism to our case of superluminal electron also requires, by the Invariant Reduction Theorem, that the field in (1.3) satisfies the matrix equation:

$$l_{1/2}\left(r; S\right) := r\left(0\right) S = \Psi_{p,\lambda,+1/2}\left(z\right) \left[= N \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \frac{c\hat{\sigma}_z p}{m_0 c^2 + \lambda E_p} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} \exp\left[i\left(pz - \lambda E_p t\right)/\hbar\right] \right]. \qquad (3.3)$$

This can happen only if, for a given $\Psi_{p,\lambda,+1/2}(z)$ there exists a 4-vector $S = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$ such that $r(0) S = \Psi_{p,\lambda,+1/2}(z)$; in other words, the 4×4 matrix $r(0)$ must be invertible. Observe that in our present case $r(0)$ is the generalized angular momentum transformation of the field [1] and is given by

$$r\left(0\right) = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{4} \\ 0 & 0 & -\frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 \end{pmatrix}.$$

Clearly, this matrix is elementary, and therefore invertible; the inverse is obtained as

$$r^{-1}\left(0\right) = \begin{pmatrix} 0 & 0 & 0 & 4 \\ 0 & 0 & 4 & 0 \\ 0 & -4 & 0 & 0 \\ -4 & 0 & 0 & 0 \end{pmatrix}.$$

Hence, the equation $r(0) S = \Psi_{p,\lambda,+1/2}(z)$ has an equilibrium solution.

Finally, it is worthwhile to notice that equations (1.4) and (1.5) of orbital and spin angular momenta shed light on the identification of antipodal points (mentioned in the previous section) which are highlighted as $x_\mu = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$ and $-x_\mu = \begin{pmatrix} -t \\ -x \\ -y \\ -z \end{pmatrix}$ under the same constant transformation of matrix

$\frac{1}{2}\sigma_{03} = \begin{pmatrix} 0 & 0 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & 0 & 0 \end{pmatrix}$, [1]. This leads to the matrix of the generalized angular momentum of the system:

$$r(0) = \begin{pmatrix} 0 & 0 & 0 & -\frac{1}{4} \\ 0 & 0 & -\frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ \frac{1}{4} & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -\cos(V/c) \\ 0 & 0 & -\cos(V/c) & 0 \\ 0 & \cos(V/c) & 0 & 0 \\ \cos(V/c) & 0 & 0 & 0 \end{pmatrix},$$

followed by the resulting derivation of the expectation value of the overall relative linear superluminal velocity component of the free spin-1/2 field:

$$V \simeq \left(\frac{21\pi}{50} + 2k\pi\right) \times c, \quad (k = 0, 1, 2, ...),$$

and this brings to an end a fiber bundle interpretation of our result.

4 CONCLUSION

The superluminal motion of free spin-half particles may be feasible in the extension of Einstein's Special Relativity to the so-called *Worm Hole Special Relativity* in *Multi-level Universes World*. The transformations underpinning these superluminal entities are identified to be the infinitesimal rotation operators of the Infinite Lorentz Group. Moreover, a fiber bundle formalism can be used to investigate this result. A principal fiber bundle with structure group $SO(3)$ and base space the two-sphere, S^2, describes static particles with spin since, one can contract Minkowski spacetime minus the world line of a particle to $\mathbb{R}^3 - \{0\}$ which, subsequently, can be retracted to S^2 sphere without changing the fiber bundle. For moving spin-1/2 particles, the underlying "unreduced" principal bundle is a product principal bundle ($T^d \times SO(3)$,p_d,T^d,R_d), with the earlier specifications. The fermion field invariance, the Decomposition Theorem, and the Invariance Reduction Theorem have all been verified. Finally, antipodal points can readily be identified together with the transformation which originates them. This makes the interpretation feasible.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

References

[1] Gazoya DKE, Oduro FT, Prempeh E. Investigation of superluminal motion of free spin-half particles in spacetime; 2015. Available: http://dx.doi.org/10.5539/jmr.v7n4p12

[2] Gerlach W, Stern O. Z. Phys. 1922;9:349.

[3] Uhlenbeck GE, Goudsmidt S. Naturw. 1925;13:953.

[4] Pauli W. Wissenschaftlicher briefwechsel mit bohr. Einstein, Heisenberg UA:Scientific Correspondence. Springer-Verlag. 1930-1939.

[5] Hill JM, Cox J. Einstein's special relativity beyond the speed of light. Proc. R. Soc. A; 2012. DOI: 10.1098/rspa.2012.0340

[6] Zhang QR. Relativity and impossibility of superluminal motion. Chin. Phys. B. 2012;21:11. Available: http://dx.doi.org/10.1088/1674-1056/21/11/110301

[7] Physical Review Letters. 2011;107:181803.

[8] John GR. Foundations of hyperbolic manifolds. Second Edition. Springer. Graduate Texts in Mathematics. 2006;149.

[9] Schrdinger E. Sitzungber. Preuss. Akad. Wiss. Phys.-Math. Kl. 1930;24:418.

[10] Dirac PAM. The quantum theory of the electron, Part 1. Proc. Roy. Soc. (London). 1928;A117:610-624. Part two. Proc. Roy. Soc. (London). 1928;A118:351-361.

[11] Greiner W. Relativistic quantum mechanics-wave equations (3rd ed.). Berlin, Heidelberg: Springer; 2000.

[12] ShaoXu R. Spin forms and spin interactions among higgs bosons, between higgs boson and graviton. Journal of Modern Physics. 2006;7:737-759. Available: http://dx.doi.org/10.4236/jmp.-2016.78070

[13] ShaoXu R. The origins of spins of elementary particles; 2014. ISBN 978-988-13649-7-5

[14] ShaoXu R. The origins of bosons and fermions. Journal of Modern Physics. Available: http:/dx.doi.org/10.4236/jmp.2014.517181

[15] ShaoXu R. Interaction of the origins of spin angular momentum 2nd edition; 2016. ISBN 978-988-14902-0-9

[16] ShaoXu R. The third kind of particles. ISBN 978-7-900500-91-5 (2011). ISBN 978-988-15598-9-0. 2012.

[17] ShaoXu R. Faster than velocity of light (Infinite Dimensional Lorentz Group of TKP); 2013. ISBN 978-988-12266-2-4

[18] Guildry M. Gauge field theories-an introduction with applications. New York: John Wiley & Sons, Inc.; 1991.

[19] Belinfante F. On the covariant derivative of tensor-undors. Physica, 1940;7(4):305-324.

[20] Ohanian HC. What Is Spin?. New York. February 5; 1984.

[21] Heinemann K, Ellison JA, Barber DP, Vogt M. New and unifying formalism for study of particle-spin dynamics using tools distilled from theory of bundles. THPRO061 in Proceedings of the 2014 International Particle Accelerator Conference, Dresden, Germany. 2014;15-20.

[22] Zimmer RJ. Ergodic theory and semisimple groups. Springer Science & Business Media. 2013;81.

[23] Feres R. Dynamical systems and semisimple groups: An introduction. Cambridge University Press, Cambridge; 1998. Handbook of dynamical systems Vol. 1A. Edited by B. Hasselblatt and A. Katok, North-Holland, Amsterdam; 2002.

[24] Ehresmann C. Les connexions infinitésimales dans un espace fibré différentiable. Colloque de Topologie (Espaces fibrés), Bruxelles. Georges Thone, Liége; Masson et Cie., Paris. 1951;29-55.

[25] Husemöller D. Fibre bundles. Springer Verlag; 1994. ISBN 0-387-94087-1

[26] Steenrod N. The topology of fibre bundles. Princeton University Press; 1951. ISBN 0-691-08055-0

[27] Bleecker D. Gauge theory and variational principles. Reading, Mass: Addison-Wesley Publishing; 1981. ISBN 0-201-10096-7

[28] Naber GL. The geometry of minkowski spacetime. New York: Springer-Verlag; 1992.

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