British Journal of Mathematics & Computer Science
4(24): 3489-3496, 2014
ISSN: 2231-0851

SCIENCEDOMAIN *international*
www.sciencedomain.org

Some Special Rogue Waves in One Type of Variable Coefficient Nonlinear Schrödinger Equations

Yue Wu[1*], Jiuli Yin[1], Xingyu Feng[1], Qianqian Xing[1] and Dan Li[1]

[1]*Nonlinear Science Research Center, Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu, 212013, P.R. China.*

Article Information

DOI: 10.9734/BJMCS/2014/12685
Editor(s):
(1) Sheng Zhang, Department of Mathematics, Bohai University, Jinzhou, China.
Reviewers:
(1) Noureddine Issaoui, Quantum Physics Laboratory, Faculty of Sciences of Monastir, University of Monastir, Monastir 5000, Tunisia.
(2) Harun-Or- Roshid, Department of Mathematics, Pabna University of Science and Technology, Bangladesh.
Complete Peer review History: http://www.sciencedomain.org/review-history.php?iid=699&id=6&aid=6365

Original Research Article

Received: 13 July 2014
Accepted: 18 August 2014
Published: 06 October 2014

Abstract

In this paper, a similarity transformation is devised to construct a Miura type transformations between one type of variable coefficient nonlinear Schrodinger equations and the cubic NLS equation. This transformation is devoted to obtain the travelling wave solutions of a complicated equation by using the solutions of a simpler equation directly. The result shows different rogue wave solutions of the variable coefficient nonlinear Schrodinger equations are given easily.

Keywords: Nonlinear schrodinger equation, similarity transformation, miura type transformation, rogue wave.

1 Introduction

Rogue waves are firstly found in the ocean. The amplitude of the rogue waves is much higher than the average wave crests around them. They are named monster waves or extreme waves, which are a threat even to large ships and ocean liners [1-5]. Rogue waves was not only found in oceans but also in optics, plasmas, superfluids and Bose-Einstein condensates. The nonlinear Schrödinger equations (NLSE) are an important model describing rogue waves. Rogue waves have been found in many different the nonlinear Schrodinger equations. Studies of finding new rogue waves of the

**Corresponding author: ywu@ujs.edu.cn;*

NLSE is an interesting work.

The inhomogeneous NLS equation with variable coefficients in the form

$$i\frac{\partial\psi}{\partial t}+\frac{\beta(t)}{2}\frac{\partial^2\psi}{\partial x^2}+v(x,t)\psi+g(t)|\psi|^2\psi=i\gamma(t)\psi \tag{1}$$

where $\psi=\psi(x,t)$, including group velocity dispersion $\beta(t)$, linear potential v(x, t), nonlinearity $g(t)$ and the gain/loss term $\gamma(t)$. When $\beta(t)=1$, Eq. (1) becomes the generalized Gross-Pitaevskii equation. Eq. (1) has been studied by many authors [6-15].

Rogue waves in an optical system has been researched in [6,7]. Furthermore, the Peregrine soliton in nonlinear fibre optics and rogue waves and rational solutions of the nonlinear Schrödinger equation were studied in [8-11]. Recently, Nonautonomous rogons in the inhomogeneous nonlinear Schrödinger equation [12] and rogue waves in the atmosphere as well as vector financial rogue waves has been demonstrated in [13-15]. Particularly, Yan used the similarity transformation and direct ansatz to obtain the analytical nonautonomous rogons of Eq. (1). In this paper, we will use a simpler method [16] to construct different rogue waves of Eq. (1). Especially, we are interested in finding some new rogue waves.

2 Similarity Transformation

In this section, we want to obtain the rogue wave solutions of Eq. (1) by a direct transformation between Eq. (1) and the famous NLS equation

$$i\frac{\partial q}{\partial X}+\frac{1}{2}\frac{\partial^2 q}{\partial T^2}+|q|^2q=0\ . \tag{2}$$

To this purpose, we make a similarity transformation as follows

$$\psi(x,t)=q(X,T)p(x,t)e^{i\phi(x,t)}, \tag{3}$$

where X is the propagation distance and T is the transverse variable. In each case, the function $\phi(x,t)$ describes the envelope of waves, and its absolute value carries information about either wave elevation, $p(x,t)$, $\phi(x,t)$ are the real-value functions, $q(X,T)$ is the complex-value functions. And, we substitute the transformation (3) into Eq. (1) and get the following system:

$$\begin{aligned}
&[ip_t - p\phi_x + \frac{\beta}{2}p_{xx} + i\beta\phi_x p_x + i\frac{\beta}{2}\phi_{xx}p - \frac{\beta}{2}\phi_x^2 p + vp - i\gamma(t)p]q \\
&+(iX_t p + \frac{\beta}{2}X_{xx}p + \beta X_x p_x + i\beta\phi_x X_x p)q_X \\
&+(iT_t p + \frac{\beta}{2}T_{xx}p + \beta T_x p_x + i\beta\phi_x T_x p)q_T \\
&+\beta T_x X_x p q_{XT} + q_{XX}\frac{\beta}{2}X_x^2 p + \frac{\beta}{2}T_x^2 p q_{TT} + gp^2|q|^2 pq = 0
\end{aligned} \tag{4}$$

Compared with Eq. (2), we have

$$\begin{cases}
gp^3 = 1 & (5)\\
\beta(t)T_x^2 p = 1 & (6)\\
X_t p + \beta(t)\phi_x X_x p = 1 & (7)\\
\frac{\beta(t)}{2}X_{xx}p + \beta(t)X_x p_x = 0 & (8)\\
\beta(t)T_x X_x p = 0 & (9)\\
\frac{\beta(t)}{2}X_x^2 p = 0 & (10)\\
T_t p + \beta(t)\phi_x T_x p = 0 & (11)\\
\frac{\beta(t)}{2}T_{xx}p + \beta(t)T_x p_x = 0 & (12)\\
p_t + \beta(t)\phi_x p_x + \frac{\beta(t)}{2}p_{xx} - \frac{\beta(t)}{2}\phi_{xx}p - \gamma(t)p = 0 & (13)\\
p\phi_x - \frac{\beta(t)}{2}p_{xx} + \frac{\beta(t)}{2}\phi_x^2 p - vp = 0 & (14)
\end{cases}$$

From (5), (8)-(10), we obtain

$$g(t) = \frac{1}{p^3},\ p = p(t),\ p \neq 0,\ X = X(t).$$

Substituting $X_x = 0$ into (7), we get $X_t = \frac{1}{p}$ and

$$X = \int \frac{1}{p}dt + x_0 . \tag{15}$$

It is noted that $p_x = 0$ for $p = p(t)$. So we deduce that $T_{xx} = 0$ from (12).Solving (6), one can have $\beta(t) = \frac{1}{T_x^2 p}$ which implies $T_x^2 \neq 0$ and $T_x = T_x(t)$. Let $T_x = f_1(t)$, we obtain

$$\beta(t) = \frac{1}{f_1^2(t)p}, \tag{16}$$

$$T = \int f_1(t)dx = f_1(t)x + f_2(t). \tag{17}$$

Substituting (16), (17) into (11), (13), (14), we obtain the following facts

$$\phi = -pf_1(t)[\frac{x^2}{2} f_1^{'}(t) + f_2^{'}(t)x] + f_3(t), \tag{18}$$

$$\gamma(t) = \frac{p_t}{p} - \frac{f_1^{'}(t)}{2f_1(t)}, \tag{19}$$

$$v = \frac{p}{2}[f_1^{'}(t)x + f_2^{'}(t)]^2 - pf_1(t)[f_1^{'}(t)x + f_2^{'}(t)], \tag{20}$$

where $f_1(t), f_2(t), f_3(t), p(t)$ are arbitrary functions.

3 Rogue Waves and Examples

The one rogue waves of Eq.(2) are given by [9,10]

$$q = (1 - \frac{4(1+2iX)}{1+4X^2+4T^2})e^{iX}, \tag{21}$$

By the previous transformation, we can obtain one-rogue wave solutions of Eq. (1)

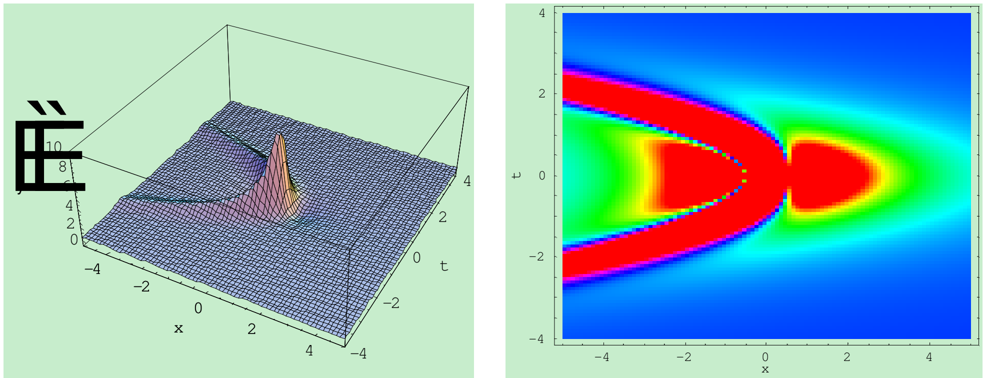
$$\psi = (1 - \frac{4(1+2iX)}{1+4X^2+4T^2})pe^{i(X+\phi)}. \tag{22}$$

The expression (22) includes different rogue wave solutions. Several examples are given as follows.

(1) Taking $p = 1, f_1(t) = 1, f_2(t) = t^2, x_0 = 0$, we have

$$|\psi|^2 = \frac{(4t^2 + 4(x+t^2)^2 - 3)^2 + (8t)^2}{(1+4t^2+4(x+t^2)^2)^2}.$$

which is shown in Fig. 1.



(a) **(b)**

Fig. 1. (a) The one-rogue wave solution and (b) the projective graph with $p=1$**,** $f_1(t)=1,\ f_2(t)=t^2, x_0=0$

(2) Taking $p=1, f_1(t)=1,, f_2(t)=t, x_0=0$, we have

$$|\psi|^2 = \frac{(4t^2+4(x+t)^2-3)^2+(8t)^2}{(1+4t^2+4(x+t)^2)^2},$$

which is shown in Fig. 2.

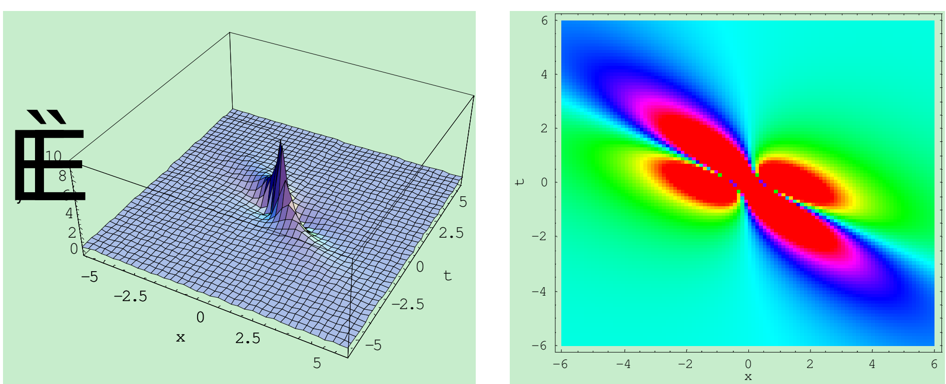


Fig. 2. (a) The one-rogue wave solution and (b) the projective graph with $p=1, f_1(t)=1,$ $, f_2(t)=t, x_0=0$

(3) Taking $p=1, f_1(t)=1, f_2(t)=t^3, x_0=0$, we have

$$|\psi|^2 = \frac{(4t^2 + 4(x+t^3)^2 - 3)^2 + (8t)^2}{(1+4t^2+4(x+t^3)^2)^2},$$

which is shown in Fig. 3.

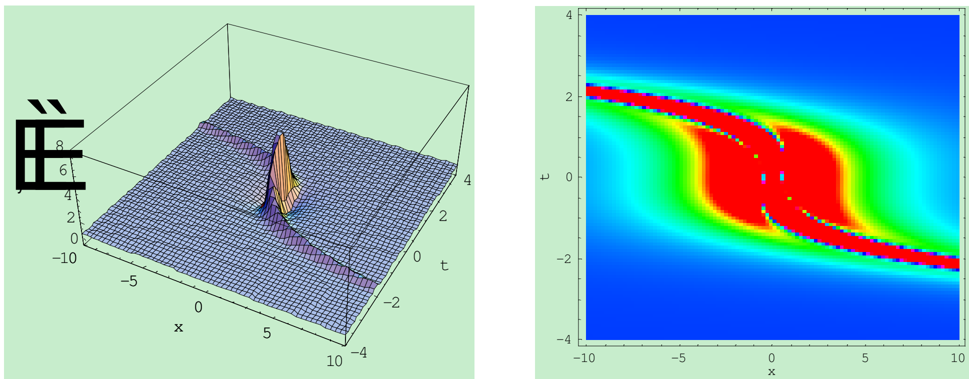


Fig. 3. (a) The one-rogue wave solution and (b) the projective graph with $p=1$, $f_1(t)=1$, $f_2(t)=t^3, x_0=0$

Another rogue waves of Eq. (2) are given by [9,10]

$$q = (1 - \frac{G+iH}{D})e^{iX},$$

where

$$G = -\frac{3}{16} + \frac{3}{2}T^2 + T^4 + \frac{9}{2}X^2 + 6T^2X^2 + 5X^4,$$

$$H = -\frac{15}{8}X - 3T^2X + 2T^4X + X^3 + 4T^2X^3 + 2X^5,$$

$$D = \frac{3}{64} + \frac{9T^2}{16} + \frac{T^4}{4} + \frac{T^6}{3} + \frac{33}{16}X^2 - \frac{3}{2}T^2X^2 + T^4X^2 + \frac{9}{4}X^4 + T^2X^4 + \frac{X^6}{3}.$$

By the similar method, we can obtain two-rogue wave solutions of Eq. (1)

$$\psi = p(1 - \frac{G+iH}{D})e^{i(X+\phi)}. \tag{23}$$

4 Conclusion

In conclusion, we construct a Miura type transformations between one type of variable coefficient nonlinear Schrödinger equations and the cubic NLS equation. The result demonstrates that different rogue wave solutions of the variable coefficient nonlinear Schrödinger equations are given easily. The presented method can also be extended to other type nonlinear Schrödinger equations.

Acknowledgements

This work is supported by the National Nature Science Foundation of China (No.11101191), and it is also sponsored by Qin Lan Project.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Dysthe K, Krogstad HE, Müller P, Oceanic Rogue Waves. Annu Rev Fluid Mech. 2008;40:287.

[2] Ruban V, Kodama Y, Ruderman M, Dudley J, Grimshaw R, Mcclintock PVE, et al. Rogue waves- towards a unifying concept. Eur Phys J Spec Top. 2010;185:5.

[3] Kharif C, Pelinovsky E. Physical mechanisms of the rogue wave phenomenon. Eur J Mech B Fluids. 2003;22:603.

[4] Müller P, Garrett CH, Osborne A. Rogue waves-The Fourteenth 'Aha Huliko'a Hawaiian Winter Workshop. Oceanography. 2005;18:66.

[5] Osborne AR. Nonlinear Ocean Waves. Academic Press, New York; 2009.

[6] Solli DR, Ropers C, Koonath P, Jalali B. Optical Rogue Waves. Nature. 2007;450:1054.

[7] Solli DR, Ropers C, Jalali B. Active Control of Rogue Waves for Stimulated Supercontinuum Generation. Phys Rev Lett. 2008;101:233-902.

[8] Kibler B, Fatome J, Finot C, Millot G, Dias F, Genty G, Akhmediev N, Dudley JM. The Peregrine soliton in nonlinear fibre optics. Nature Phys. 2010;6:1.

[9] Akhmediev N, Ankiewicz A, Soto-Crespo JM. Rogue waves and rational solutions of the nonlinear Schrödinger equation. Phys Rev E. 2009;80:026601.

[10] Akhmediev N, Ankiewicz A, Taki M. Waves that appear from nowhere and disappear without a trace. Phys Lett A. 2009;373-675.

[11] Yu V. Bludov, Konotop VV, Akhmediev N. Rogue waves as spatial energy concentrators in arrays of nonlinear waveguides. Opt Lett. 2009;34:3015.

[12] Yan ZY. Nonautonomous "rogons" in the inhomogeneous nonlinear Schrödinger equation with variable coefficients. Phys Lett A. 2010;374:672.

[13] Stenflo L, Marklund M, Plasma J. Rogue waves in the atmosphere. Phys. 2010;76:293.

[14] Yan ZY. Financial Rogue Waves. Commun Theor Phys. 2010;54:947.

[15] Yan ZY. Vector financial rogue waves. Phys Lett A. 2011;375:4274. Zhenya Yan, Vector financial rogue waves. Phys Lett A. 2011;375:4274.

[16] He JS, Li Y. Designable Integrability of the Variable Coefficient Nonlinear Schrödinger Equations 2010 Stud. Appl Math. 2010;126:1.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
www.sciencedomain.org/review-history.php?iid=699&id=6&aid=6365