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A Heuristic Algorithm for Solving Two Person Zero Sum Game Using Triangular Fuzzy Number

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Authors' contributions

This work was carried out in collaboration between all authors. Authors GKS and HKD initiated and designed the study and provided the techniques behind the algorithm and illustrations of it and wrote the first draft of the manuscript. Author Kamrunnahar provided some important background of the study and managed the literature searches. All authors read and approved the final manuscript.

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Abstract

The present study deals with two person zero sum game problem and fuzzy linear programming (FLP) problem, and then development of an algorithm using FLP. To accomplish this goal, the two person zero sum game problems were first converted into linear programming (LP) problems and then by using FLP the solution of the problem was obtained. The new method is a systematic procedure and can be utilised for all types of two person zero sum game problem irrespective of maximise or minimise objective function. At the end, this method is illustrated with a number of numerical examples.

Keywords: Game theory; fuzzy linear programming problem; triangular fuzzy number; membership function; fuzzy version of simplex method.

1 Introduction

Fuzzy set theory has become an important tool in the branch of decision making sciences and has been applied successfully in too many disciplines such as control theory, management sciences, and industrial

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applications as well as in the field of decision making process in fuzzy environment. To moderate these problems, researchers of various disciplines developed many mathematical models using fuzzy technique in which membership functions, introduced by Zadeh [1], have been used. As uncertainty and vagueness have become the essential part of our lives, therefore, it is impossible to eliminate all sorts of uncertainties from the living world. It also refers to those sectors or organisations, which mainly deal with manufacturing and production management. Therefore, it is evident that in the real world, decisions are taken on the basis of information which, at least in part, is fuzzy in nature. That is why fuzzy techniques are used as a useful tool to deal with those situations which cannot be handled with classical methods.

Bellman and Zadeh [2] proposed the concept of decision making in fuzzy environments. After the revolutionary work on fuzzy linear programming by Tanaka [3] and Zimmermann [4], several kinds of fuzzy linear programming problems have appeared in the literature and different methods have been proposed to solve such problems. In fuzzy decision making problems, the concept of maximising decision was proposed by Bellman and Zadeh [2]. Zimmermann [5] presented a fuzzy approach to multi-objective linear programming problems. The above methods can solve the fuzzy linear programming problems which are fuzzy in nature. Moreover, those methods are not capable of solving fuzzy linear programming problem completely. Fuzzy version of simplex method is suitable to obtain an optimum solution of fuzzy linear programming problems fully.

John von Neumann, one of the greatest mathematicians of the 20th century is most closely associated with the creation of the theory of games. Although others preceded him in formulating a theory of games - notably ´Emile Borel - it was von Neumann, who published a paper in 1928 that laid the foundation for the theory of two-person zero-sum games. Von Neumann's work culminated in his fundamental book on game theory. To solve two-persons zero-sum game, its formulation by linear programming problem needs to be done. In this paper, two-person zero-sum game problems have been solved, which involved linear programming problem, by conversion of the problem as FLP problem. These problems were solved using fuzzy version of simplex method. Rest of the paper is organised as follows: Section II describes preliminaries and properties of two person zero sum game and some basic operations for FLP problems. Section III describes algorithm and its illustrations on the two person zero sum games by using FLP. In Section IV, the conclusion of this paper has been presented.

2 Preliminaries and Properties

There are a number of methods such as graphical method, algebraic method, method of sub-games, matrix method, iterative method, dominance method for solving two persons zero sum game problems with their merits and demerits. The detailed discussions of the theory of games relevant for present purposes may be found in the works of other researchers [6-9]. In the current section, we considered the $m \times n$ two-person zero-sum LP game and some relevent topics on FLP.

2.1 Discussion of $m \times n$ game

Any two person zero sum game with mixed strategies can be solved by transforming the problem to a L.P. Let the value of the game is v. Initially, player I acts as maximise and player II acts as minimise. But after transforming some steps the LP is converted resulting inversed value of the game . For this object function also changes.

Consider the optimal mixed strategy for player II, Expected payoff for player II $= \sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij} y_j x_i$ and the player II strategy $(x_1, x_2, \ldots, x_n)$ is optimal if $\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij} y_j x_i \le v$ for all opposing strategies. i.e., player I is $(y_1, y_2, \ldots, y_n)$. Finally, the following forms for the player II and I are obtained respectively.

Player II:

Maximise, $\frac{1}{v} = x_1 + x_2 + \ldots\ldots + x_m$

Subject to,

$$\left.\begin{array}{ccccccccc} p_{11}x_1 & + & p_{12}x_2 & + & \ldots & + & p_{1n}x_n & \le & 1 \\ p_{21}x_1 & + & p_{22}x_2 & + & \ldots & + & p_{2n}x_n & \le & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ p_{m1}x_1 & + & p_{m2}x_2 & + & \ldots & + & p_{mn}x_n & \le & 1 \\ x_1 & + & x_2 & + & \ldots & + & x_n & = & 1 \end{array}\right\} \quad (1)$$

and $x_j \ge 0$, for $j = 1, 2, \ldots, n$.

Player I:

Minimise $\frac{1}{v} = y_1 + y_2 + \ldots\ldots + y_m$

Subject to,

$$\left.\begin{array}{ccccccccc} p_{11}y_1 & + & p_{21}y_2 & + & \ldots & + & p_{m1}y_m & \ge & 1 \\ p_{12}y_1 & + & p_{22}y_2 & + & \ldots & + & p_{m2}y_m & \ge & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ p_{1n}y_1 & + & p_{2n}y_2 & + & \ldots & + & p_{mn}y_m & \ge & 1 \\ y_1 & + & y_2 & + & \ldots & + & y_m & = & 1 \end{array}\right\} \quad (2)$$

and $y_i \ge 0$, for $i = 1, 2, \ldots, m$.

Equations (1) and (2) can be solved by suitable L.P. method such as usual simplex method or Big M simplex method or Primal-dual simplex method., The present work solves (1) and (2) by using fuzzy version of simplex method.

2.2 FLP problem

A FLP [10,11], in which all parameters and variables are considered as triangular fuzzy numbers, is formulated as follows:

$$\max\ Z \approx \sum_{j=1}^{n} C_j X_j$$

$$\text{Subject to } \sum_{j=1}^{n} A_{ij} X_j \le B_i \quad (i \in N_m)$$

$$X_j \ge 0 \quad (j \in N_n)$$

where A_{ij} , B_i , C_j are fuzzy numbers and X_j are variables whose states are fuzzy numbers $(i \in N_m, j \in N_n)$; the operations of additions and multiplications are operations of fuzzy arithmetic; and $\leq$, $\geq$ denotes the ordering of fuzzy numbers. Instead of discussing this general type, the issues involved by two special cases of fuzzy linear programming problems have been exemplified here.

2.3 Triangular membership function

A fuzzy set $\tilde{A}$, is called triangular fuzzy number with Peak (or Core) a_2, left width $a_1 > 0$ and right width $a_3 > 0$ if its membership function has the following form

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{a_2 - x}{a_1} \ , & \text{if } a_2 - a_1 \leq x \leq a_2 \\ 1 - \dfrac{x - a_2}{a_3} \ , & \text{if } a_2 \leq x \leq a_2 + a_3 \\ 0 \ , & \text{Otherwise} \end{cases}$$

and the set of all triangular fuzzy numbers is denoted by $R(\mathscr{F})$ where $\tilde{A}$ in parametric form is

$\tilde{A} = \left(a_1(r-1) + a_2, a_3(1-r) + a_2\right), r \in [0, 1]$.

Symbolically, $\tilde{A} = (a_1, a_2, a_3)$.

2.4 Distance between two ordered triples

Let $\tilde{A}$ and $\tilde{B}$ be two ordered triples where $\tilde{A} = (a_1, a_2, a_3)$, $\tilde{B} = (b_1, b_2, b_3)$ and $r \in [0, 1]$, then the signed distance of $\tilde{A}$ and $\tilde{B}$ is as follows:

$$\begin{aligned} d(\tilde{A}, \tilde{B}) &= \int_0^1 \left[M\left(\tilde{A}(r)\right) - M\left(\tilde{B}(r)\right) \right] dr \\ &= \frac{1}{4}[4(a_2 - b_2) - (a_1 - b_1) + (a_3 - b_3)]. \end{aligned}$$

2.5 Zero ordered triples

An ordered triples $\tilde{A} = (a_1, a_2, a_3)$ is said to be zero and only if $4a_2 - a_1 + a_3 = 0$, or an ordered triples $\tilde{A}$ is said to be zero if $Core(\tilde{A}) = 0$. It is to be noted that $\tilde{0}$ is equivalent to $(0,0,0)$ and the signed distance of ordered triples $\tilde{A} = (a_1, a_2, a_3)$ is $d(\tilde{A}, \tilde{0}) = \dfrac{1}{4}(4a_2 - a_1 + a_3)$.

2.6 Operations on ordered triples

Let $\tilde{A}$ and $\tilde{B}$ be two ordered triples [11, 12] where $\tilde{A}=(a_1,a_2,a_3)$, $\tilde{B}=(b_1,b_2,b_3)$. Then

i) $\tilde{A}+\tilde{B}=(a_1,a_2,a_3)+(b_1,b_2,b_3)=(a_1+b_1,a_2+b_2,a_3+b_3);$

ii) $-\tilde{B}=-(b_1,b_2,b_3)=(b_3,-b_2,b_1);$

iii) $\tilde{A}-\tilde{B}=(a_1,a_2,a_3)-(b_1,b_2,b_3)=(a_1+b_3,a_2-b_2,a_3+b_1);$

iv) $\tilde{A}\cdot\tilde{B}=(a_1,a_2,a_3)\cdot(b_1,b_2,b_3)=(a_2b_1+b_2a_1,a_2b_2,a_2b_3+b_2a_3)$, if $\tilde{A}$ and $\tilde{B}$ are positive;

v) $\tilde{A}\cdot\tilde{B}=(a_3,-a_2,a_1)\cdot(b_1,b_2,b_3)=(b_2a_3-a_2b_1,-a_2b_2,b_2a_1-a_2b_3)$, if $\tilde{A}$ is negative and $\tilde{B}$ is positive;

vi) $\tilde{A}\cdot\tilde{B}=(a_3,-a_2,a_1)\cdot(b_3,-b_2,b_1)=(-a_2b_3-b_2a_3,a_2b_2,-a_2b_1-b_2a_1)$, if $\tilde{A}$ and $\tilde{B}$ are negative;

vii) $\tilde{A}^{-1}=\dfrac{1}{\tilde{A}}=\left(\dfrac{1}{a_1},\dfrac{1}{a_2},\dfrac{1}{a_3}\right);$

viii) $\tilde{A}/\tilde{B}=(a_1,a_2,a_3)/(b_1,b_2,b_3)=(a_2/b_1+a_1/b_2,a_2/b_2,a_2/b_3+a_3/b_2)$, if $\tilde{A}$ and $\tilde{B}$ are positive;

ix) $\tilde{A}/\tilde{B}=(a_3,-a_2,a_1)/(b_1,b_2,b_3)=(a_3/b_2-a_2/b_1,-a_2/b_2,a_1/b_2-a_2/b_3)$, if $\tilde{A}$ is negative and $\tilde{B}$ is positive;

x) $\tilde{A}/\tilde{B}=(a_3,-a_2,a_1)/(b_3,-b_2,b_1)=(-a_2/b_3-a_3/b_2,a_2/b_2,-a_2/b_1-a_1/b_2)$, if $\tilde{A}$ and $\tilde{B}$ are negative;

xi) $$\lambda\tilde{A}=\begin{cases}(\lambda a_1,\lambda a_2,\lambda a_3) & ,\text{ if }\lambda>0\\(-\lambda a_1,-\lambda a_2,-\lambda a_3) & ,\text{ if }\lambda<0\end{cases}$$

3 Main Results

An algorithm for solving two person zero sum game problem was introduced at the end, and this algorithm was illustrated with a number of numerical examples.The main results of this work are depending on algorithms and problems [6-9,13,14,12,15].

3.1 Algorithm

This Algorithm was incorporated with the fuzzy version of simplex method to solve zero sum game problems. To do this, the generalized $m\times n$ game was first considered as a pay-off matrix, and then to find the strategies of two players FLP by Simplex algorithm was used, the existence of an initial basic feasible fuzzy solution was always assumed. The steps are as follows:

Step (1) :Consider the pay-off matrix to find the strategies of two players and the game value.

Step (2) : Search the minimum element from each row of the payoff matrix and then find the maximum element of these minimum elements.

Step (3) **:** Search the maximum element from each column of the payoff matrix and then find the minimum element of these maximum elements

Step (4) **:** For the player I if the Maximin is less than zero then find k which is equal to addition of one and absolute value of Maximin.

Step (5) : For the player II if the Minimax is less than zero then find k which is equal to addition of one and absolute value of Minimax.

Step (6) : If Maximin and Minimax both are greater than zero then, $k \geq 0$.

Step (7)**:** Finally to get the modified payoff matrix adding k with each payoff elements of the given payoff matrix.

Step (8) : Then to find the mixed strategies for player II, the modified payoff matrix was converted into equation (1) otherwise consider equation (2) for player I. For doing this, follow the following steps:

Step (9) **:** Check whether the objective function of the given FLP is to be minimised or maximised. If it is to be minimised then it was converted into a problem of maximising as follows:

$$\text{Minimize}\,\tilde{Z} = -\text{Maximize}\,(-\tilde{Z})$$

Substep 1: Check whether all $\tilde{b}_i\,(i = 1, 2, \ldots, m)$ are non-negative. If any one of $\tilde{b}_i$ is negative then multiply the corresponding inequation of the constant by -1 , so as to get all $\tilde{b}_i\,(i = 1, 2, \ldots, m)$ non-negative.

Substep 2: Convert all the inequalities of the constraints into equations by introducing slack and or surplus fuzzy variables in the constraints. Put the costs of these variables equal to zero.

Substep 3: Obtain an initial basic feasible solution of the problem in the form $\tilde{X}_B \approx \tilde{B}^{-1}\tilde{b}$ and put in the first column of the Simplex table.

Substep 4: Compute the net evaluations $\tilde{z}_j - \tilde{c}_j\,(j = 1, 2, \ldots, n)$ by using the relation $\tilde{z}_j - \tilde{c}_j = \tilde{c}_B \tilde{y}_j - \tilde{c}_j$, where any column $\tilde{A}_j \approx \sum_{i=1}^{m} \tilde{y}_{ij}\tilde{b}_i \approx \tilde{y}_{1j}\tilde{b}_1 + \tilde{y}_{2j}\tilde{b}_2 + \ldots + \tilde{y}_{mj}\tilde{b}_m \approx \tilde{y}_j \tilde{B}.$

Examine the sign of $\tilde{z}_j - \tilde{c}_j$:

i) If all $\left(\tilde{z}_j - \tilde{c}_j\right) \geq 0$ then the initial basic feasible fuzzy solution $\tilde{x}_B$ is an optimum basic feasible fuzzy solution.

ii) If at least one $\left(\tilde{z}_j - \tilde{c}_j\right) < 0$, proceed on to the next step.

Substep 5: If there are more than one negative $\tilde{z}_j - \tilde{c}_j$, then choose the most negative of them. Let it be $\tilde{z}_r - \tilde{c}_r$ for some $j = r$.

i) If all $\tilde{y}_{ir} \le 0\,(i = 1, 2, \ldots, m)$, then there is an unbounded solution to the given problem.

ii) If at least one $\tilde{y}_{ir} > 0\,(i = 1, 2, \ldots, m)$, then the corresponding vector $\tilde{y}_r$ enter the basis $\tilde{y}_B$.

Substep 6: Compute $\dfrac{\tilde{x}_{B_i}}{\tilde{y}_{ir}}, i = 1, 2, \ldots, m$ and choose minimum of them. Let minimum of these ratios be $\dfrac{\tilde{x}_{B_r}}{\tilde{y}_{kr}}$.

Then the vector $\tilde{y}_k$ will level the basis $\tilde{y}_B$. The common element $\tilde{y}_{kr}$, which in the k^{th} row and r^{th} column is known as leading triangular fuzzy number of the table.

Substep 7: Convert the leading triangular fuzzy number to unit triangular fuzzy number by dividing its row by the leading triangular fuzzy number itself and all other elements in its column to zero triangular fuzzy number by making use of relation:

$$\tilde{y} \approx \tilde{y}_{ij} - \left(\tilde{y}_{kj} / \tilde{y}_{kr}\right)\tilde{y}_{ir}, i = 1, 2, \ldots, m+1; i \ne k \text{ and } \tilde{y}_{kj} \approx \tilde{y}_{kj} / \tilde{y}_{kr}, j = 0, 1, 2, \ldots, n.$$

Substep 8: Go to substep 4 and repeat the computational procedure until either an optimum solution is obtained or these is an indication of an unbounded solution.

3.2 Illustrative and numerical examples

Problem-1: Consider the game with the following pay-off matrix. Find the game value with their strategies. The problem is taken from Taha [16].

$$\begin{array}{cc} & \textit{Player II} \\ \textit{Player I} & \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix} \end{array}$$

Solution:

To find the modified payoff matrix for player II, search the maximum element from each column of the payoff matrix and then find the minimum element of these maximum elements for Minimax. Here Minimax is greater than zero i.e. $k \ge 0$ so the given payoff matrix would be the modified payoff matrix. Now, let the value of the game and the strategies for player II be v and $y_i; i = 1, 2$ respectively. Now by setting, $\dfrac{y_i}{v} = Y_i; i = 1, 2$; $\dfrac{1}{v} = V$ and using the standard procedure for solving two person zero sum game of LP discussed in the references [6-9], the following LP problem for player II was obtained.

$$\text{Maximise}: \frac{1}{v} = Y_1 + Y_2$$

$$\text{Subject to} \quad \begin{aligned} -2Y_1 + 3Y_2 &\le 1 \\ 3Y_1 - 4Y_2 &\le 1 \\ Y_1, Y_2 &\ge 0 \end{aligned}$$

If we convert the above LP into fuzzy LP problem, we obtain the following:

$$\text{Maximise } \tilde{Z} \approx (1,1,1)\tilde{y}_1 + (0.5,1,0.5)\tilde{y}_2$$

$$\text{s/t:} \quad \begin{aligned} (-1,-2,1)\tilde{y}_1 + (1,3,2)\tilde{y}_2 &\le (1,1,1) \\ (1,3,2)\tilde{y}_1 + (1,-4,1)\tilde{y}_2 &\le (1,1,1) \\ \tilde{y}_1, \tilde{y}_2 &\ge 0 \end{aligned}$$

For the solution of any FLP by fuzzy version of Simplex algorithm, the existence of an initial basic feasible fuzzy solution is always assumed. First of all the inequalities of the constraints were converted into equations by introducing slack and/or surplus fuzzy variables in the constraints and put the costs of these variables equal to zero to convert the problem to standard form as follows:

$$\text{Maximise } \tilde{Z} \approx (1,1,1)\tilde{y}_1 + (0.5,1,0.5)\tilde{y}_2 + (0,0,0)\tilde{s}_1 + (0,0,0)\tilde{s}_2$$

$$\text{s/t:} \quad \begin{aligned} (-1,-2,1)\tilde{y}_1 + (1,3,2)\tilde{y}_2 + (0,1,0)\tilde{s}_1 \phantom{+ (0,1,0)\tilde{s}_2} &= (1,1,1) \\ (1,3,2)\tilde{y}_1 + (1,-4,1)\tilde{y}_2 \phantom{+ (0,1,0)\tilde{s}_1} + (0,1,0)\tilde{s}_2 &= (1,1,1) \\ \tilde{y}_1, \tilde{y}_2, \tilde{s}_1, \tilde{s}_2 &\ge 0 \end{aligned}$$

Table 1. Simplex table

		$(1,1,1)$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$(0,0,0)$	$(0,0,0)$	
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{x}_{B_i}$
$(0,0,0)$	$\tilde{s}_1$	$(-1,-2,1)$	$(1,3,2)$*	$(0,1,0)$	$(0,0,0)$	$(1,1,1)$
$(0,0,0)$	$\tilde{s}_2$	$(1,3,2)$	$(1,-4,1)$	$(0,0,0)$	$(0,1,0)$	$(1,1,1)$
		$(1,-1,1)$	$(½,-1,½)$	$(0,0,0)$	$(0,0,0)$	

Table 2. Simplex table

		$(1,1,1)$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$(0,0,0)$	$(0,0,0)$	
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{x}_{B_i}$
$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$\tilde{y}_2$	$\left(-2,-\frac{2}{3},-\frac{2}{3}\right)$	$\left(\frac{10}{3},1,\frac{13}{6}\right)$	$\left(1,\frac{1}{3},\frac{1}{2}\right)$	$(0,0,0)$	$\left(\frac{4}{3},\frac{1}{3},\frac{5}{6}\right)$
$(0,0,0)$	$\tilde{s}_2$	$\left(-\frac{23}{3},\frac{1}{3},-\frac{4}{3}\right)$*	$\left(\frac{46}{3},0,\frac{32}{3}\right)$	$\left(\frac{13}{6},\frac{4}{3},\frac{7}{3}\right)$	$(0,1,0)$	$\left(\frac{20}{3},\frac{7}{3},\frac{4}{3}\right)$
		$\left(-\frac{4}{3},-\frac{5}{3},0\right)$	$\left(\frac{13}{3},0,\frac{19}{6}\right)$	$\left(\frac{7}{6},\frac{1}{3},\frac{2}{3}\right)$	$(0,0,0)$	

Table 3. Simplex table

		$(1,1,1)$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$(0,0,0)$	$(0,0,0)$	
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{x}_{B_i}$
$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$\tilde{y}_2$	$\left(-\frac{135}{9},0,-\frac{25}{9}\right)$	$\left(\frac{42}{3},1,\frac{141}{6}\right)$	$\left(\frac{107}{9},3,\frac{133}{9}\right)$	$\left(\frac{5}{3},2,\frac{5}{3}\right)$	$\left(\frac{31}{3},5,\frac{217}{18}\right)$
$(1,1,1)$	$\tilde{y}_1$	$\left(-\frac{68}{3},1,-\frac{11}{3}\right)$	$(46,0,32)$	$\left(\frac{43}{3},4,\frac{25}{3}\right)$	$(1,3,1)$	$\left(10,7,\frac{40}{3}\right)$
		$\left(-\frac{321}{9},0,-\frac{40}{9}\right)$	$\left(\frac{183}{3},0,\frac{337}{6}\right)$	$\left(\frac{571}{18},7,\frac{247}{9}\right)$	$\left(\frac{20}{3},5,\frac{20}{3}\right)$	

So have the optimal solution $\tilde{y}_1=\left(10,7,\frac{40}{3}\right)$ and $\tilde{y}_2=\left(\frac{31}{3},5,\frac{217}{18}\right)$ and the optimal value $\tilde{Z}_{\max}\approx(1,1,1)\left(10,7,\frac{40}{3}\right)+\left(\frac{1}{2},1,\frac{1}{2}\right)\left(\frac{31}{3},5,\frac{217}{18}\right)=\left(\frac{179}{6},12,\frac{314}{9}\right)$ in the fuzzy version. Finally, $\frac{1}{v}=\left(\frac{179}{6},12,\frac{314}{9}\right)$ and so the value of the game for the modified matrix is $v=\left(\frac{6}{179},\frac{1}{12},\frac{9}{314}\right)$. Then, the corresponding crisp version is following: Player II's strategies $\left(\frac{7}{12},\frac{5}{12}\right)$ and the game value of $\frac{1}{12}$.

Similarly, to find the modified payoff matrix for the player I, search the minimum element from each row of the payoff matrix and then find the maximum element of these minimum elements for Maximin. Here Maximin is greater than zero. Then $k\geq 0$ so the given payoff matrix would be the modified payoff matrix. Now, for player I, let the value of the game and the strategies be v and $x_i\,;i=1,2$ respectively. By setting, $\frac{x_i}{v}=X_i;i=1,2$; $\frac{1}{v}=V$ and using the standard procedure for solving two person zero sum game of LP discussed in the references [6-9], the following LP problem for player I was obtained.

$$\text{Minimise } \frac{1}{v}=X_1+X_2$$

$$\begin{aligned}-2X_1+3X_2&\geq 1\\ 3X_1-4X_2&\geq 1\end{aligned}$$

$$X_1,X_2\geq 0$$

If we convert the above LP into fuzzy LP problem, we obtain the following:

$$\text{Minimise } \tilde{Z}\approx(1,1,1)\tilde{x}_1+(1,1,1)\tilde{x}_2$$

$$\text{s/t:}\quad \begin{array}{l} (-1,-2,-1)\tilde{x}_1 + (2,3,2)\tilde{x}_2 \geq (1,1,1) \\ (2,3,2)\tilde{x}_1 + (-2,-4,-2)\tilde{x}_2 \geq (1,1,1) \\ \tilde{x}_1, \tilde{x}_2 \geq 0 \end{array}$$

Now, converting the above minimizing problem into maximising problem as below:

$$\text{Maximise } \tilde{Z}^* \approx (1,1,1)\tilde{x}_1 + (1,1,1)\tilde{x}_2$$

$$\text{s/t:}\quad \begin{array}{l} (-1,-2,-1)\tilde{x}_1 + (2,3,2)\tilde{x}_2 \leq (1,1,1) \\ (2,3,2)\tilde{x}_1 + (-2,-4,-2)\tilde{x}_2 \leq (1,1,1) \\ \tilde{x}_1, \tilde{x}_2 \geq 0 \end{array}$$

For the solution of any FLPP by fuzzy version of Simplex algorithm, the existence of an initial basic feasible fuzzy solution was always assumed. First, all the inequalities of the constraints were converted into equations by introducing slack and/ or surplus fuzzy variables in the constraints and the costs of these variables equal to zero was put to convert the problem to standard form as follows:

$$\text{Maximise } \tilde{Z}^* \approx (1,1,1)\tilde{y}_1 + (1,1,1)\tilde{y}_2 + (0,0,0)\tilde{s}_1 + (0,0,0)\tilde{s}_2$$

$$\text{s/t:}\quad \begin{array}{l} (-1,-2,-1)\tilde{x}_1 + (2,3,2)\tilde{x}_2 + (0,1,0)\tilde{s}_1 = (1,1,1) \\ (2,3,2)\tilde{x}_1 + (-2,-4,-2)\tilde{x}_2 + (0,1,0)\tilde{s}_2 = (1,1,1) \\ \tilde{x}_1, \tilde{x}_2, \tilde{s}_1, \tilde{s}_2 \geq 0 \end{array}$$

Table 4. Simplex table

		$(1,1,1)$	$(1,1,1)$	$(0,0,0)$	$(0,0,0)$	
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{x}_{B_i}$
$(0,0,0)$	$\tilde{s}_1$	$(-1,-2,-1)$	$(2,3,2)^*$	$(0,1,0)$	$(0,0,0)$	$(1,1,1)$
$(0,0,0)$	$\tilde{s}_2$	$(2,3,2)$	$(-2,-4,-2)$	$(0,0,0)$	$(0,1,0)$	$(1,1,1)$
		$(1,-1,1)$	$(1,-1,1)$	$(0,0,0)$	$(0,0,0)$	

Table 5. Simplex table

		$(1,1,1)$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$(0,0,0)$	$(0,0,0)$	
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{x}_{B_i}$
$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$\tilde{s}_1$	$\left(\frac{13}{3},0,\frac{13}{3}\right)$	$\left(-\frac{14}{3},\frac{1}{3},-\frac{14}{3}\right)^*$	$(0,1,0)$	$\left(\frac{4}{3},\frac{2}{3},\frac{4}{3}\right)$	$\left(3,\frac{5}{3},3\right)$
$(1,1,1)$	$\tilde{y}_1$	$\left(\frac{13}{6},1,\frac{13}{6}\right)$	$\left(-\frac{8}{3},-\frac{4}{3},-\frac{8}{3}\right)$	$(0,0,0)$	$\left(\frac{1}{2},\frac{1}{3},\frac{1}{2}\right)$	$\left(\frac{5}{6},\frac{1}{3},\frac{5}{6}\right)$
		$\left(\frac{25}{6},0,\frac{25}{6}\right)$	$\left(-3,-\frac{7}{3},-3\right)$	$(0,0,0)$	$\left(\frac{5}{6},\frac{1}{3},\frac{5}{6}\right)$	

Table 6. Simplex table

		$(1,1,1)$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$(0,0,0)$	$(0,0,0)$	
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{x}_{B_i}$
$(1,1,1)$	$\tilde{y}_2$	$(13,0,13)$	$\left(-\frac{41}{3},1,-\frac{41}{6}\right)$	$(1,3,1)$	$\left(\frac{14}{3},2,\frac{14}{3}\right)$	$\left(\frac{32}{3},5,\frac{32}{3}\right)$
$(1,1,1)$	$\tilde{y}_1$	$\left(\frac{117}{3},1,\frac{117}{6}\right)$	$(-21,0,-21)$	$(1,4,1)$	$\left(\frac{39}{6},3,\frac{39}{6}\right)$	$\left(\frac{87}{6},7,\frac{87}{6}\right)$
		$\left(\frac{207}{6},0,\frac{207}{6}\right)$	$\left(-\frac{98}{3},0,-\frac{98}{3}\right)$	$(9,7,9)$	$\left(\frac{97}{6},5,\frac{97}{6}\right)$	

So have the optimal solution $\tilde{x}_1=\left(\frac{87}{6},7,\frac{87}{6}\right)$ and $\tilde{x}_2=\left(\frac{32}{3},5,\frac{32}{3}\right)$ and the optimal value $\tilde{Z}_{\max}\approx(1,1,1)\left(\frac{87}{6},7,\frac{87}{6}\right)+(1,1,1)\left(\frac{32}{3},5,\frac{32}{3}\right)=\left(\frac{223}{6},12,\frac{223}{6}\right)$ in the fuzzy version. Finally, $\frac{1}{v}=\left(\frac{223}{6},12,\frac{223}{6}\right)$ and so the value of the game for the modified matrix is $v=\left(\frac{6}{223},\frac{1}{12},\frac{6}{223}\right)$. Then, the corresponding crisp version is following: Player I's strategies $\left(\frac{7}{12},\frac{5}{12}\right)$ and the game value of $\frac{1}{12}$.

Problem-2: Two oil companies, USA Oil Co. and Caltex, operating in a city, are trying to increase their market at the expense of the other. The USA Oil Co. is considering possibilities of decreasing price, giving free soft drinks on Rs. 40 purchases of oil or giving away a drinking glass with each 40 litter purchase. Obviously, Caltex cannot ignore this and comes out with its own program to increase its share in the market. The payoff matrix forms in the viewpoints of increasing or decreasing market shares is given in table below.The problem is taken from Dhar and Das [17], Lieberman [18] and Taha [16].

Table 7. 3×3 payoff matrix games

Caltex / USA	I	II	III
1	1%	1%	4%
2	4%	-2%	4%
3	-1%	2%	-2%

Solution:

The optimum strategies for players and the value of the game is to be addressed

$$\begin{array}{cc} & \textit{Player II} \\ \textit{Player I} & \begin{bmatrix} 1 & 1 & 4 \\ 4 & -2 & 4 \\ -1 & 2 & -2 \end{bmatrix} \end{array}$$

To find the modified payoff matrix for player II, search the maximum element from each column of the payoff matrix and then find the minimum element of these maximum elements for Minimax. Here Minimax is greater than zero i.e. $k \geq 0$ so the given payoff matrix would be the modified payoff matrix. Now, let the value of the game and the strategies for player II be v and $y_i; i = 1,2,3$ respectively. Now by setting, $\frac{y_i}{v} = Y_i, i = 1,2,3$, $\frac{1}{v} = V$ and using the standard procedure for solving two person zero sum game of LP discussed in the references [6-9], the following LP problem for player II was obtained.

$$\text{Maximise } \frac{1}{v} = Y_1 + Y_2 + Y_3$$

$$\text{s/t:} \quad \begin{aligned} Y_1 + Y_2 + 4Y_3 &\leq 1 \\ 4Y_1 - 2Y_2 + 4Y_3 &\leq 1 \\ -Y_1 + 2Y_2 - 2Y_3 &\leq 1 \end{aligned}$$

$$Y_j \geq 0, \text{ for } j = 1,2,3.$$

By converting the above LP into fuzzy LP problem, the following was obtained:

$$\text{Maximise } \tilde{Z} \approx (1,1,1)\tilde{y}_1 + (1,1,1)\tilde{y}_2 + (1,1,1)\tilde{y}_3$$

$$\text{s/t:} \quad \begin{aligned} \left(\frac{1}{2},1,\frac{1}{2}\right)\tilde{y}_1 + (1,1,1)\tilde{y}_2 + (2,4,2)\tilde{y}_3 &\leq \left(\frac{1}{2},1,\frac{1}{2}\right) \\ (2,4,2)\tilde{y}_1 + (1,-2,1)\tilde{y}_2 + (1,4,1)\tilde{y}_3 &\leq \left(\frac{1}{2},1,\frac{1}{2}\right) \\ (1,-1,1)\tilde{y}_1 + (1,2,1)\tilde{y}_2 + (1,-2,1)\tilde{y}_3 &\leq (1,1,1) \\ \tilde{y}_1, \tilde{y}_2, \tilde{y}_3 &\geq 0 \end{aligned}$$

Solution:

For the solution of any FLPP by fuzzy version of Simplex algorithm, the existence of an initial basic feasible fuzzy solution is always assumed. First of all the inequalities of the constraints was converted into equations by introducing slack and/ or surplus fuzzy variables in the constraints and the costs of these variables was put equal to zero to convert the problem to standard form as follows:

$$\text{Maximise } \tilde{Z} \approx (1,1,1)\tilde{y}_1 + (1,1,1)\tilde{y}_2 + (1,1,1)\tilde{y}_3 + (0,0,0)\tilde{s}_1 + (0,0,0)\tilde{s}_2 + (0,0,0)\tilde{s}_3$$

$$\text{s/t:} \quad \begin{aligned} \left(\frac{1}{2},1,\frac{1}{2}\right)\tilde{y}_1 + (1,1,1)\tilde{y}_2 + (2,4,2)\tilde{y}_3 + (0,1,0)\tilde{s}_1 &= \left(\frac{1}{2},1,\frac{1}{2}\right) \\ (2,4,2)\tilde{y}_1 + (1,-2,1)\tilde{y}_2 + (1,4,1)\tilde{y}_3 + (0,1,0)\tilde{s}_2 &= \left(\frac{1}{2},1,\frac{1}{2}\right) \\ (1,-1,1)\tilde{y}_1 + (1,2,1)\tilde{y}_2 + (1,-2,1)\tilde{y}_3 + (0,1,0)\tilde{s}_3 &= (1,1,1) \\ \tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \tilde{s}_1, \tilde{s}_2, \tilde{s}_3 &\geq 0 \end{aligned}$$

Table 8. Simplex table

		$(1,1,1)$	$(1,1,1)$	$(1,1,1)$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	$\tilde{x}_{B_i}$
$(0,0,0)$	$\tilde{s}_1$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$(1,1,1)$	$(2,4,2)$	$(0,1,0)$	$(0,0,0)$	$(0,0,0)$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$
$(0,0,0)$	$\tilde{s}_2$	$(2,4,2)^*$	$(1,-2,1)$	$(1,4,1)$	$(0,0,0)$	$(0,1,0)$	$(0,0,0)$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$
$(0,0,0)$	$\tilde{s}_3$	$(1,-1,1)$	$(1,2,1)$	$(1,-2,1)$	$(0,0,0)$	$(0,0,0)$	$(0,1,0)$	$(1,1,1)$
		$(1,-1,1)$	$(1,-1,1)$	$(1,-1,1)$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	

Table 9. Simplex table

		$(1,1,1)$	$(1,1,1)$	$(1,1,1)$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	$\tilde{x}_{B_i}$
$(0,0,0)$	$\tilde{s}_1$	$(-1,0,-1)$	$\left(\frac{5}{4},\frac{3}{2},\frac{5}{4}\right)^*$	$\left(\frac{3}{4},3,\frac{3}{4}\right)$	$(0,1,0)$	$\left(-\frac{1}{4},-\frac{1}{4},-\frac{1}{4}\right)$	$(0,0,0)$	$\left(\frac{1}{8},\frac{3}{4},\frac{1}{8}\right)$
$(1,1,1)$	$\tilde{y}_1$	$\left(\frac{5}{2},1,\frac{5}{2}\right)$	$\left(-\frac{3}{4},-\frac{1}{2},-\frac{3}{4}\right)$	$\left(\frac{9}{4},1,\frac{9}{4}\right)$	$(0,0,0)$	$\left(\frac{1}{2},\frac{1}{4},\frac{1}{2}\right)$	$(0,0,0)$	$\left(\frac{5}{8},\frac{1}{4},\frac{5}{8}\right)$
$(0,0,0)$	$\tilde{s}_3$	$\left(\frac{7}{2},0,\frac{7}{2}\right)$	$\left(\frac{1}{4},\frac{3}{2},\frac{1}{4}\right)$	$\left(\frac{13}{4},-1,\frac{13}{4}\right)$	$(0,0,0)$	$\left(\frac{1}{2},\frac{1}{4},\frac{1}{2}\right)$	$(0,1,0)$	$\left(\frac{13}{8},\frac{5}{4},\frac{13}{8}\right)$
		$\left(\frac{9}{2},0,\frac{9}{2}\right)$	$\left(-\frac{1}{4},-\frac{3}{2},-\frac{1}{4}\right)$	$\left(\frac{17}{4},0,\frac{17}{4}\right)$	$(0,0,0)$	$\left(\frac{3}{4},\frac{1}{4},\frac{3}{4}\right)$	$(0,0,0)$	

Table 10. Simplex table

		$(1,1,1)$	$(1,1,1)$	$(1,1,1)$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	
$\tilde{c}_B$	$\tilde{y}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	$\tilde{x}_{B_i}$
$(1,1,1)$	$\tilde{y}_2$	$\left(-\frac{2}{3},0,-\frac{2}{3}\right)$	$\left(\frac{61}{30},1,\frac{61}{30}\right)$	$\left(\frac{29}{10},2,\frac{29}{10}\right)$	$\left(\frac{9}{5},\frac{2}{3},\frac{9}{5}\right)$	$\left(-\frac{11}{30},-\frac{1}{6},-\frac{11}{30}\right)$	$(0,0,0)$	$\left(\frac{41}{60},\frac{1}{2},\frac{41}{60}\right)$
$(1,1,1)$	$\tilde{y}_1$	$\left(\frac{13}{6},1,\frac{13}{6}\right)$	$\left(\frac{23}{30},0,\frac{23}{30}\right)$	$\left(\frac{47}{10},2,\frac{47}{10}\right)$	$\left(\frac{11}{15},\frac{1}{3},\frac{11}{15}\right)$	$\left(\frac{7}{30},\frac{1}{6},\frac{7}{30}\right)$	$(0,0,0)$	$\left(\frac{73}{60},\frac{1}{2},\frac{73}{60}\right)$
$(0,0,0)$	$\tilde{s}_3$	$\left(\frac{5}{2},0,\frac{5}{2}\right)$	$\left(\frac{3}{2},0,\frac{3}{2}\right)$	$(4,-4,4)$	$(0,-1,0)$	$\left(\frac{1}{4},\frac{1}{2},\frac{1}{4}\right)$	$(0,1,0)$	$\left(\frac{7}{4},\frac{1}{2},\frac{7}{4}\right)$
		$\left(\frac{3}{2},0,\frac{3}{2}\right)$	$\left(\frac{171}{30},0,\frac{171}{30}\right)$	$\left(\frac{63}{5},3,\frac{63}{5}\right)$	$\left(\frac{53}{15},1,\frac{53}{15}\right)$	$\left(\frac{1}{5},\frac{1}{3},\frac{1}{5}\right)$	$(0,0,0)$	

So the optimal solution $\tilde{y}_1 = \left(\frac{73}{60}, \frac{1}{2}, \frac{73}{60}\right)$, $\tilde{y}_2 = \left(\frac{41}{60}, \frac{1}{2}, \frac{41}{60}\right)$ and $\tilde{y}_3 = (0,0,0)$ and the optimal value $\tilde{Z}_{\max} \approx (1,1,1)\left(\frac{73}{60}, \frac{1}{2}, \frac{73}{60}\right) + (1,1,1)\left(\frac{41}{60}, \frac{1}{2}, \frac{41}{60}\right) + (1,1,1)(0,0,0) = \left(\frac{174}{60}, 1, \frac{174}{60}\right)$ in the fuzzy version. Finally, $\frac{1}{v} = \left(\frac{174}{60}, 1, \frac{174}{60}\right)$ was obtained and so the value of the game for the modified matrix is $v = \left(\frac{60}{174}, 1, \frac{60}{174}\right)$. Then, the corresponding crisp version is following: Player I's strategies $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$ and the game value 1.

Similarly, to find the modified payoff matrix for the player I, search the minimum element from each row of the payoff matrix and then find the maximum element of these minimum elements for Maximin. Here Maximin is greater than zero then $k \geq 0$ so the given payoff matrix would be the modified payoff matrix. Now, for player I let the value of the game and the strategies be v and $x_i; i = 1,2,3$ respectively. Now by setting, $\frac{x_i}{v} = Y_i$, $\frac{1}{v} = V$ and using the standard procedure for solving two person zero sum game of LP discussed in the references [6-9], the following LP problem for player I was obtained.

$$\text{Minimise } \frac{1}{v} = X_1 + X_2 + X_3$$

$$\text{s/t:} \quad \begin{array}{rcrcrcl} X_1 & + & 4X_2 & - & X_3 & \geq & 1 \\ X_1 & - & 2X_2 & + & 2X_3 & \geq & 1 \\ 4X_1 & + & 4X_2 & - & 2X_3 & \geq & 1 \end{array}$$

$$X_i \geq 0, \text{ for } i = 1,2,3.$$

by converting the above LP into fuzzy LP problem, the following was obtained:

$$\text{Minimise } \tilde{Z} \approx (0.5,1,0.5)\tilde{x}_1 + (0.5,1,0.5)\tilde{x}_2 + (0.5,1,0.5)\tilde{x}_3$$

$$\text{s/t:} \quad \begin{array}{l} \left(\frac{1}{2},1,\frac{1}{2}\right)\tilde{x}_1 + (2,4,2)\tilde{x}_2 + (-2,-1,-2)\tilde{x}_3 \geq \left(\frac{1}{2},1,\frac{1}{2}\right) \\ \left(\frac{1}{2},1,\frac{1}{2}\right)\tilde{x}_1 + (-1,-2,-1)\tilde{x}_2 + (1,2,1)\tilde{x}_3 \geq \left(\frac{1}{2},1,\frac{1}{2}\right) \\ (2,4,2)\tilde{x}_1 + (2,4,2)\tilde{x}_2 + (-3,-2,-3)\tilde{x}_3 \geq \left(\frac{1}{2},1,\frac{1}{2}\right) \\ \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq 0 \end{array}$$

Solution:

Converting the above minimising problem into maximising problem as below:

$$\text{Maximise } \tilde{Z}^* \approx (0.5,1,0.5)\tilde{x}_1 + (0.5,1,0.5)\tilde{x}_2 + (0.5,1,0.5)\tilde{x}_3$$

$$\text{s/t:}\quad \begin{array}{l} \left(\frac{1}{2},1,\frac{1}{2}\right)\tilde{x}_1 + \left(\frac{1}{2},1,\frac{1}{2}\right)\tilde{x}_2 + (2,4,2)\tilde{x}_3 \le \left(\frac{1}{2},1,\frac{1}{2}\right) \\ (2,4,2)\tilde{x}_1 + (-1,-2,-1)\tilde{x}_2 + (2,4,2)\tilde{x}_3 \le \left(\frac{1}{2},1,\frac{1}{2}\right) \\ (-2,-1,-2)\tilde{x}_1 + (1,2,1)\tilde{x}_2 + (-3,-2,-3)\tilde{x}_3 \le \left(\frac{1}{2},1,\frac{1}{2}\right) \\ \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ge 0 \end{array}$$

For the solution of any FLPP by fuzzy version of Simplex algorithm, the existence of an initial basic feasible fuzzy solution was always assumed. First of all the inequalities of the constraints was converted into equations by introducing slack and/or surplus fuzzy variables in the constraints and the costs of these variables was put equal to zero to convert the problem to standard form as follows:

Maximise $\tilde{Z} \approx (0.5,1,0.5)\tilde{x}_1 + (0.5,1,0.5)\tilde{x}_2 + (0.5,1,0.5)\tilde{x}_3 + (0,0,0)\tilde{s}_1 + (0,0,0)\tilde{s}_2 + (0,0,0)\tilde{s}_3$

$$\text{s/t:}\quad \begin{array}{l} \left(\frac{1}{2},1,\frac{1}{2}\right)\tilde{x}_1 + \left(\frac{1}{2},1,\frac{1}{2}\right)\tilde{x}_2 + (2,4,2)\tilde{x}_3 + (0,1,0)\tilde{s}_1 = \left(\frac{1}{2},1,\frac{1}{2}\right) \\ (2,4,2)\tilde{x}_1 + (-1,-2,-1)\tilde{x}_2 + (2,4,2)\tilde{x}_3 + (0,1,0)\tilde{s}_2 = \left(\frac{1}{2},1,\frac{1}{2}\right) \\ (-2,-1,-2)\tilde{x}_1 + (1,2,1)\tilde{x}_2 + (-3,-2,-3)\tilde{x}_3 + (0,1,0)\tilde{s}_3 = \left(\frac{1}{2},1,\frac{1}{2}\right) \\ \tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{s}_1, \tilde{s}_2, \tilde{s}_3 \ge 0 \end{array}$$

Table 11. Simplex table

		$(0.5,1,0.5)$	$(0.5,1,0.5)$	$(0.5,1,0.5)$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	
$\tilde{c}_B$	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	$\tilde{y}_{B_i}$
$(0,0,0)$	$\tilde{s}_1$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$(2,4,2)$	$(0,1,0)$	$(0,0,0)$	$(0,0,0)$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$
$(0,0,0)$	$\tilde{s}_2$	$(2,4,2)^*$	$(-1,-2,-1)$	$(2,4,2)$	$(0,0,0)$	$(0,1,0)$	$(0,0,0)$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$
$(0,0,0)$	$\tilde{s}_3$	$(-3,-1,-2)$	$(1,2,1)$	$(-3,-2,-3)$	$(0,0,0)$	$(0,0,0)$	$(0,1,0)$	$\left(\frac{1}{2},1,\frac{1}{2}\right)$
		$\left(\frac{1}{2},-1,-\frac{1}{2}\right)$	$\left(\frac{1}{2},-1,-\frac{1}{2}\right)$	$\left(\frac{1}{2},-1,-\frac{1}{2}\right)$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	

Table 12. Simplex table

		(0.5,1,0.5)	(0.5,1,0.5)	(0.5,1,0.5)	(0,0,0)	(0,0,0)	(0,0,0)	
$\tilde{c}_B$	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	$\tilde{y}_{B_i}$
$(0,0,0)$	$\tilde{s}_1$	$(3,0,3)$	$\left(-\frac{3}{4},\frac{3}{2},-\frac{3}{4}\right)^*$	$\left(\frac{9}{2},3,\frac{9}{2}\right)$	$(0,1,0)$	$\left(\frac{1}{2},-\frac{1}{4},\frac{1}{2}\right)$	$(0,0,0)$	$\left(\frac{9}{8},\frac{3}{4},\frac{9}{8}\right)$
$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$\tilde{x}_1$	$\left(\frac{5}{2},1,\frac{5}{2}\right)$	$\left(-\frac{5}{4},-\frac{1}{2},-\frac{5}{4}\right)$	$\left(\frac{5}{2},1,\frac{5}{2}\right)$	$(0,0,0)$	$\left(\frac{1}{2},\frac{1}{4},\frac{1}{2}\right)$	$(0,0,0)$	$\left(\frac{5}{8},\frac{1}{4},\frac{5}{8}\right)$
$(0,0,0)$	$\tilde{s}_3$	$\left(\frac{1}{2},0,\frac{1}{2}\right)$	$\left(-\frac{1}{4},\frac{3}{2},-\frac{1}{4}\right)$	$\left(-\frac{1}{2},-1,-\frac{1}{2}\right)$	$(0,0,0)$	$\left(\frac{1}{2},\frac{1}{4},\frac{1}{2}\right)$	$(0,1,0)$	$\left(\frac{9}{8},\frac{5}{4},\frac{9}{8}\right)$
		$\left(\frac{7}{2},0,\frac{7}{2}\right)$	$\left(-1,-\frac{3}{2},-1\right)$	$\left(\frac{7}{2},0,\frac{7}{2}\right)$	$(0,0,0)$	$\left(\frac{5}{8},\frac{1}{4},\frac{5}{8}\right)$	$(0,0,0)$	

Table 13. Simplex table

		(0.5,1,0.5)	(0.5,1,0.5)	(0.5,1,0.5)	(0,0,0)	(0,0,0)	(0,0,0)	
$\tilde{c}_B$	$\tilde{x}_B$	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	$\tilde{y}_{B_i}$
$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$\tilde{x}_2$	$(2,0,2)$	$\left(-\frac{5}{2},1,-\frac{5}{2}\right)$	$(-12,2,-12)$	$\left(-\frac{4}{3},\frac{2}{3},-\frac{4}{3}\right)$	$\left(\frac{2}{3},-\frac{1}{6},\frac{2}{3}\right)$	$(0,0,0)$	$\left(-\frac{1}{4},\frac{1}{2},-\frac{1}{4}\right)$
$\left(\frac{1}{2},1,\frac{1}{2}\right)$	$\tilde{x}_1$	$\left(\frac{9}{2},1,\frac{9}{2}\right)$	$\left(-\frac{15}{4},0,-\frac{15}{4}\right)$	$\left(-\frac{19}{2},2,-\frac{19}{2}\right)$	$\left(-\frac{4}{3},-\frac{1}{3},-\frac{4}{3}\right)$	$\left(\frac{7}{6},\frac{1}{6},\frac{7}{6}\right)$	$(0,0,0)$	$\left(\frac{3}{8},\frac{1}{2},\frac{3}{8}\right)$
$(0,0,0)$	$\tilde{s}_3$	$\left(\frac{7}{2},0,\frac{7}{2}\right)$	$(-1,0,-1)$	$(4,-4,4)$	$(0,-1,0)$	$\left(1,\frac{1}{2},1\right)$	$(0,1,0)$	$\left(\frac{9}{4},\frac{1}{2},\frac{9}{4}\right)$
		$(7,0,7)$	$\left(\frac{21}{4},0,\frac{21}{4}\right)$	$(-19,3,-19)$	$\left(-\frac{13}{6},1,-\frac{13}{6}\right)$	$\left(\frac{11}{6},0,\frac{11}{6}\right)$	$(0,0,0)$	

So the optimal solution $\tilde{x}_1 = \left(\frac{3}{8}, \frac{1}{2}, \frac{3}{8}\right)$, $\tilde{x}_2 = \left(-\frac{1}{4}, \frac{1}{2}, -\frac{1}{4}\right)$ and $\tilde{x}_3 = (0, 0, 0)$ and the optimal value $\tilde{Z}_{max} \approx (0.5, 1, 0.5)\left(\frac{3}{8}, \frac{1}{2}, \frac{3}{8}\right) + (0.5, 1, 0.5)\left(-\frac{1}{4}, \frac{1}{2}, -\frac{1}{4}\right) + (0.5, 1, 0.5)(0, 0, 0) = \left(\frac{5}{8}, 1, \frac{5}{8}\right)$ in the fuzzy version. Finally, $\frac{1}{v} = \left(\frac{5}{8}, 1, \frac{5}{8}\right)$ was obtained and so the value of the game for the modified matrix was $v = \left(\frac{8}{5}, 1, \frac{8}{5}\right)$. Then, the corresponding crisp version was following: Player I's strategies $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$ and the game value 1.

4 Conclusion

In this paper, an algorithm for solving two person zero sum game by using fuzzy version of simplex method with non-negative Fuzzy Technical coefficients was introduced. Two person zero sum game problem as crisp linear programming problem was formulated and hence it was solved by using fuzzy version of simplex method. To illustrate the algorithm, a number of numerical examples were demonstrated. To solve game problem in fuzzy environment, it would be great idea for readers and the present approach will help anyone to solve game problem in fuzzy environment.

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Competing Interests

Authors have declared that no competing interests exist.

References

[1] Zadeh LA. Fuzzy sets. Information and Control. 1965;8:338–353.

[2] Bellman RE, Zadeh LA. Decision making in a fuzzy environment. Management Sci. 1970;17:141–164.

[3] Tanaka H, Asai K. Fuzzy linear programming problems with fuzzy numbers. Fuzzy Sets and Systems. 1984;13:1–10.

[4] Zimmermann HJ. Optimization in fuzzy environment. Presented at XXI Int. TIMES and 46th ORSA Conference, San Juan, Puerto Rioco; 1974.

[5] Zimmermman HJ. Application of fuzzy set theory to mathematical programming. Information Sciences. 1985;36:29-58.

[6] Das HK, Saha T, Babul Hasan M. Numerical experiments by improving a numerical methods for solving game problems through computer algebra. International Journal of Decision Sciences. 2012; 3(1):23-52.

[7] Das HK. A comparative study on two person zero sum game problems through computer algebra. Dhaka University Journal of Science. 2017;65(1):1-8.

[8] Das HK, Sajal Chakroborty. Analyzing on the decomposition based pricing procedure for solving two person zero sum game problems through computer algebra. International Journal of Computer Applications. 2016;156(12):37-47.

[9] Das HK, Babul Hasan M. An algorithm and its computer technique for solving game problems using LP method. Int. J. of Basic & Applied Sciences. 2011;11(3):90-99.

[10] Carlsson C, Korhonen P. A parametric approach to fuzzy linear programming. Fuzzy Sets and Systems. 1986;20:17-30.

[11] Gani AN, Duraisamy C, Veeramani C. A note on fuzzy linear programming problem using L-R fuzzy number. International Journal of Algorithms, Computing and Mathematics. 2009;2(3):93–106.

[12] Shirin S, Kamrunnahar. Application of fuzzy optimization problem in fuzzy environment. Dhaka University Journal of Science. 2014;62(2):119-125.

[13] Klir GJ, Yuan B. Fuzzy sets and fuzzy logic theory and applications. Prentice-Hall of India Private Limited, New Delhi. 1995;1-117,408–415.

[14] Kumar A, Singh P. A new method for solving fully fuzzy linear programming problems. Annals of Fuzzy Mathematics and Informatics. 2012;3(1):103–118.

[15] Shih TS, Su JS, Yao JS. Fuzzy linear programming based on interval-valued fuzzy sets. International Journal of Innovative Computing, Information and Control. 2009;5(8):2081–2090.

[16] Taha HA. Operation research: An introduction. 6th edition. Prentice-Hall of India Private Limited, New Delhi; 1997.

[17] Abir Sutra Dharand, Das HK. Analyzing decomposition procedures in LP and unraveling for the two persons zero sum game and transportation problem. 20th International Math Conference, Bangladesh Mathematical Society. 2017;13:24-25.

[18] Hillier FS, Lieberman GJ. Introduction to operation research. 8th edition. Tata McGraw Hill Education Private Limited, New Delhi; 2010.

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