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Global Solvability of a Degenerate Diffusion Equation with Time Delay

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

In this paper, we study the existence of global solution for a time delayed p-Laplace equation. By means of fixed point approach, we showed the existence and uniqueness of global weak solution.

Keywords: p-Laplace; time delay; schaefer's fixed point method.

1 Introduction

In this paper, we consider the existence of solutions for a time delayed equation of the following p-Laplace equation with time delay

$$\frac{\partial u}{\partial t} - \Delta_p u = a|u|^{q_1-1}u + b|u_\tau|^{q_2-1}u_\tau, \qquad (x,t) \in Q, \tag{1.1}$$

with initial and boundary value conditions

$$u(x,t) = 0, \qquad (x,t) \in \partial\Omega \times (0,\infty), \tag{1.2}$$

$$u(x,t) = \eta(x,t), \qquad (x,t) \in Q_{-\tau}, \tag{1.3}$$

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where the p-Laplace operator is defined as $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ with $p > 1$; $Q = \Omega \times (0, \infty)$, $Q_{-\tau} = \Omega \times [-\tau, 0]$, $\Omega \subset \mathbb{R}^N$ is a bounded and smooth enough domain; $0 \le q_1 \le 1$, $0 \le q_2 \le 1$, $u_\tau(x,t) = u(x, t-\tau)$, a, b and $\tau > 0$ are constants; and $\eta_0(x) \in L^2(\Omega) \cap W^{1,p}(\Omega)$, $\eta(x,t) \in W_2^{1,1}(Q_{-\tau})$, and $\nabla\eta \in L^p(Q_{-\tau})$.

As an important class of partial differential equations, the time delayed diffusion equations come from a variety of physical phenomena appeared widely in nature, which have been treated by many investigators for years, and various methods have been proposed to study different properties of the problem, such as the existence and uniqueness of solutions [1, 2, 3], traveling wave solutions[4, 5], asymptotic behavior[6, 7], etc. Most of these discussions in the literature are devoted to linear and semilinear parabolic equations, with the time delays occur in the reaction functions, but for the quasilinear parabolic equation with time delay, as far as we know, there are very limited works have been done [4, 5, 8]. In this paper, we shall study the existence of solutions for the initial and boundary value problem of a degenerate diffusion equation as (1.1) with the time delay occur on the nonlinear source term.

We aimed to study the existence of global solutions for the above problem, and we give the main result as follows

Theorem 1.1. *Assume that $q_1, q_2 \in [0,1]$, then for any constants a, b, the initial and boundary value problem (1.1)-(1.3) admits a unique global solution $u \in E$. Furthermore, if $a = 0$, $b \neq 0$, then for any $q_2 \ge 0$, the solution will exists globally for any initial data.*

2 Preliminary

Before going further, we first give the definition of weak solutions

Definition 2.1. A function $u \in E$ is called weak solution of the initial and boundary value problem (1.1)-(1.3), if and only if the following equalities hold

$$\iint_{Q_T} \frac{\partial u}{\partial t}\varphi dxdt + \iint_{Q_T} |\nabla u|^{p-2}\nabla u \nabla\varphi dxdt = \iint_{Q_T} (a|u|^{q_1-1}u + b|u_\tau|^{q_2-1}u_\tau)\varphi(x,t)dxdt, \qquad \forall\ \varphi \in \mathring{C}^\infty(Q_T), \tag{2.1}$$

and

$$\lim_{t\to 0^+} \int_\Omega u(x,t)h(x)dx = \int_\Omega \eta(x,0)h(x)dx, \quad \text{for } \forall\ h \in C_0^\infty(\Omega), \tag{2.2}$$

$$u(x,t) = \eta(x,t), \qquad \text{for } (x,t) \in \Omega \times [-\tau, 0]. \tag{2.3}$$

Here, $E = \{u \in L^2(Q_T);\ \frac{\partial u}{\partial t} \in L^2(Q_T), u \in W_0^{1,p}(Q_T)\}$.

In the following section, we are going to prove the existence of generalized solutions by using the Schaefer's Fixed Point Theorem. For reader's convenience, we give this theorem, it is stated as follows [9]

Theorem 2.1. (Schaefer's Fixed Point Theorem) *X denote a real Banach space, Suppose*

$$A : X \to X$$

is a continuous and compact mapping. Assume further that the set

$$\{u \in X; u = \lambda A[u] \text{ for some } 0 \le \lambda \le 1\}$$

is bounded. Then A has a fixed point.

3 The Existence of Generalized Solutions

First of all, we need to construct a completely continuous mapping, set

$$\Phi(v) = a|v|^{q_1-1}v + b|v_\tau|^{q_2-1}v_\tau.$$

For any $T > 0$, clearly, if $v \in L^2(\tilde{Q}_T)$, then $\Phi(v) \in L^2(Q_T)$. Denote

$$X = \{u; u \in L^2(\tilde{Q}_T), \text{and } u|_{\partial\Omega}(x,t) = 0, \text{ for } t > 0\},$$

where $\tilde{Q}_T = \Omega \times (-\tau, T)$, $Q_T = \Omega \times (0, T)$. It's not difficult to verify that X is a Banach space.

Define a mapping

$$F(\Phi(\cdot)) : X \to X,$$
$$F(\Phi(v)) = u,$$

where, u is a generalized solution of the following system

$$\begin{aligned} &\frac{\partial u}{\partial t} - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = \Phi(v), && (x,t) \in Q_T, \\ &u(x,t) = 0, && (x,t) \in \partial\Omega \times (0,T), \\ &u(x,t) = \eta(x,t), && (x,t) \in Q_{-\tau}. \end{aligned}$$

It is not difficult to see that for any $v \in X$, $\Phi(v) \in L^2(Q_T)$, then the above problem admits a unique generalized solution $u \in E$ by the standard theory of p-Laplace equations.

Lemma 3.1. *The mapping $F(\Phi(\cdot))$ is continuous.*

Proof. Assume $\{v_k\} \subset X$, $v \in X$ and $v_k \to v$ in the sense of $\|\cdot\|_{L^2(Q_T)}$. In addition, suppose $u_k = F(\Phi(v_k))$, $u = F(\Phi(v))$, and let $\omega_k = u_k - u \in E$. Obviously, we have $\omega_k = 0$, when $t \le 0$. Then we have

$$\iint_{Q_T} \frac{\partial w_k}{\partial t}\varphi dxdt + \iint_{Q_T} (|\nabla u_k|^{p-2}\nabla u_k - |\nabla u|^{p-2}\nabla u)\nabla\varphi dxdt = \iint_{Q_T} (\Phi(v_k) - \Phi(v))\,\varphi dxdt, \quad \forall\ \varphi \in \mathring{C}^\infty(Q_T), \tag{3.1}$$

by the density of $\mathring{C}^\infty(Q_T)$ in E, for any given $t > 0$, we may choose $\varphi = \omega_k \cdot \chi_{[0,t]}(s)$, where, $\chi_{[0,t]}$ is the characteristic function on $[0, t]$, then we obtain

$$\begin{aligned} \frac{1}{2}\iint_{Q_t} \frac{\partial w_k^2}{\partial t} dxds + \iint_{Q_t} (|\nabla u_k|^{p-2}\nabla u_k - |\nabla u|^{p-2}\nabla u)\nabla\omega_k dxds &= \iint_{Q_t} (\Phi(v_k) - \Phi(v))\,\omega_k dxds \\ &\le \frac{1}{2}\iint_{Q_t} (\Phi(v_k) - \Phi(v))^2 dxds + \frac{1}{2}\iint_{Q_t} \omega_k^2 dxds, \end{aligned} \tag{3.2}$$

by further computation, we obtain

$$\frac{1}{2}\int_\Omega w_k^2(x,t)dx + \iint_{Q_t} (|\nabla u_k|^{p-2}\nabla u_k - |\nabla u|^{p-2}\nabla u)\nabla\omega_k dxds \le \frac{1}{2}\iint_{Q_t} (\Phi(v_k) - \Phi(v))^2 dxdt + \frac{1}{2}\iint_{Q_t} \omega_k^2 dxdt,$$

according to the monotone property of $|\nabla u|^{p-2}\nabla u$, and combining with Gronwall's inequality, yield

$$\int_\Omega w_k^2(x,t)dx \le \iint_{Q_t} (\Phi(v_k)-\Phi(v))^2 dxdt \cdot e^t, \tag{3.3}$$

which means

$$\iint_{Q_T} w_k^2(x,t)dxdt \le \iint_{Q_T} (\Phi(v_k)-\Phi(v))^2 dxdt \cdot (e^T-1). \tag{3.4}$$

Noticing

$$\begin{aligned}
&\iint_{Q_T} (\Phi(v_k)-\Phi(v))^2 dxdt \\
\le& 2a^2 \iint_{Q_T} (|v_k|^{q_1-1}v_k - |v|^{q_1-1}v)^2 dxdt + 2b^2 \iint_{Q_T} (|v_{k\tau}|^{q_2-1}v_{k\tau} - |v_\tau|^{q_2-1}v_\tau)^2 dxdt \\
\le& 2a^2 \left(\iint_{Q_T} (|v_k|^{q_1-1}v_k - |v|^{q_1-1}v)^{\frac{2}{q_1}}\right)^{q_1} |Q_T|^{1-q_1} \\
&+ 2b^2 \left(\iint_{Q_T} (|v_{k\tau}|^{q_2-1}v_{k\tau} - |v_\tau|^{q_2-1}v_\tau)^{\frac{2}{q_2}} dxdt\right)^{q_2} |Q_T|^{1-q_2}.
\end{aligned}$$

Here, $|Q_T|$ represents the Lebesgue measure of Q_T. Because $v_k \to v$ in $L^2(Q_T)$, therefore we have

$$|v_k|^{q_1-1}v_k \to |v|^{q_1-1}v \quad \text{in } L^{2/q_1}(Q_T);$$

and

$$|v_{k\tau}|^{q_2-1}v_{k\tau} \to |v_\tau|^{q_2-1}v_\tau, \quad \text{in } L^{2/q_2}(Q_T).$$

combining with the inequality above, we deduce

$$\Phi(v_k) \to \Phi(v), \qquad \text{in } L^2(Q_T).$$

In view of (3.4), and combining with $\omega_k = 0$, when $t \le 0$, we infer $\omega_k \to 0$ in the sense of $L^2(Q_T)$, namely $u_k \to u$ in $L^2(Q_T)$, which implies that $F(\Phi(\cdot))$ is a continuous mapping. □

Lemma 3.2. *The mapping $F(\Phi(\cdot))$ is compact.*

Proof. Assume $\{v_k\} \subset X$, and there exists a constant $M > 0$, such that

$$\|v_k\|_{L^2(Q_T)} \le M.$$

Then

$$\begin{aligned}
\|\Phi(v_k)\|^2_{L^2(Q_T)} &\le 2a^2 \iint_{Q_T} |v_k|^{2q_1} dxdt + 2b^2 \iint_{Q_T} |v_{k\tau}|^{2q_2} dxdt \\
&\le 2a^2 \left(\iint_{Q_T} v_k^2 dxdt\right)^{q_1} |Q_T|^{1-q_1} + 2b^2 \left(\iint_{Q_T} v_{k\tau}^2 dxdt\right)^{q_2} |Q_T|^{1-q_2} \\
&\le 2a^2 M^{2q_1} |Q_T|^{1-q_1} + 2b^2 M^{2q_2} |Q_T|^{1-q_2} \\
&= \widetilde{M}.
\end{aligned}$$

For the initial and boundary value problem of p-Laplace equation of the following form

$$\begin{aligned}
&\frac{\partial u}{\partial t} - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = f(x,t), && \text{for } (x,t) \in \Omega \times (0,T], \\
&u(x,t) = 0, && \text{for } (x,t) \in \partial\Omega \times (0,T], \\
&u(x,0) = \eta_0(x), && \text{for } x \in \Omega,
\end{aligned}$$

where, $f \in L^2(Q_T)$ and $\eta_0(x) \in W^{1,p}(\Omega)\bigcap L^2(\Omega)$.

By the standard theory of the p-Laplace equations, we have estimates as follows

$$\iint_{Q_T} u^2 dxdt \le \left(\iint_{Q_T} f^2 dxdt + \int_\Omega \eta_0^2 dx\right)(e^T - 1), \tag{3.5}$$

$$\iint_{Q_T} |\nabla u|^p dxdt \le \left(\iint_{Q_T} f^2 dxdt + \int_\Omega \eta_0^2 dx\right)e^T, \tag{3.6}$$

$$\iint_{Q_T} |\frac{\partial u}{\partial t}|^2 dxdt \le C\left(\iint_{Q_T} f^2(x,t)dxdt + \frac{2}{p}\int_\Omega (|\nabla\eta_0(x)|^2+1)^{p/2}dx\right). \tag{3.7}$$

Here, C merely depends on p and Ω. Applying these estimates above to u_k, we further obtain

$$\begin{aligned}\iint_{Q_T} u_k^2 dxdt &\le \left(\iint_{Q_T} \Phi(v_k(x,t))^2 dxdt + \int_\Omega \eta_0^2 dx\right)(e^T-1) + \iint_{Q_{-\tau}} \eta^2(x,t)dxdt\\ &\le \left(\widetilde{M} + \int_\Omega \eta_0^2 dx\right)(e^T-1) + \iint_{Q_{-\tau}} \eta^2(x,t)dxdt,\end{aligned} \tag{3.8}$$

$$\begin{aligned}\iint_{Q_T} |\nabla u_k|^p dxdt &\le \left(\iint_{Q_T} \Phi(v_k(x,t))^2 dxdt + \int_\Omega \eta_0^2 dx\right)e^T + \iint_{Q_{-\tau}} |\nabla\eta|^p(x,t)dxdt\\ &\le \left(\widetilde{M} + \int_\Omega \eta_0^2 dx\right)e^T + \iint_{Q_{-\tau}} |\nabla\eta|^p(x,t)dxdt,\end{aligned} \tag{3.9}$$

$$\begin{aligned}\iint_{Q_T} |\frac{\partial u_k}{\partial t}|^2 dxdt &\le C\left(\iint_{Q_T} \Phi(v_k(x,t))^2 dxdt + \frac{2}{p}\int_\Omega (|\nabla\eta_0(x)|^2+1)^{p/2}dx\right) + \iint_{Q_{-\tau}} |\frac{\partial\eta}{\partial t}|^2 dxdt\\ &\le C\left(\widetilde{M} + \frac{2}{p}\int_\Omega (|\nabla\eta_0(x)|^2+1)^{p/2}dx\right) + \iint_{Q_{-\tau}} |\frac{\partial\eta}{\partial t}|^2 dxdt.\end{aligned} \tag{3.10}$$

From the above estimates, we can see that u_k, $\frac{\partial u_k}{\partial t}$ is bounded uniformly in $L^2(Q_T)$, and ∇u_k is bounded uniformly in $L^p(Q_T)$. From the compact embedding theorem, we derive u_k is compact in $L^2(Q_T)$. That is $F(\Phi(\cdot))$ is a compact mapping. The proof is complete. □

Proof of Theorem 1.1. Now, we have showed that the mapping $F(\Phi(\cdot))$ is continuous and compact. In order to apply Schaefer's Fixed Point Theorem to show the existence of solutions, it suffices to prove the boundedness of the set

$$\{u \in X; u = \lambda F(\Phi(u)) \text{ for some } 0 \le \lambda \le 1\}.$$

Assume $u = \lambda F(\Phi(u))$, $0 \le \lambda \le 1$. Then if $\lambda = 0$, then

$$u = 0;$$

While if $\lambda \ne 0$, then

$$\begin{aligned}&\iint_{Q_T} \frac{\partial u}{\partial t}\varphi dxdt + \lambda^{2-p}\iint_{Q_T} |\nabla u|^{p-2}\nabla u\nabla\varphi dxdt\\ &= \lambda\iint_{Q_T} (a|u|^{q_1-1}u + b|u_\tau|^{q_2-1}u_\tau)\varphi(x,t)dxdt, \qquad \forall\ \varphi \in \mathring{C}^\infty(Q_T),\\ &u(x,t) = 0, \qquad \text{for } (x,t) \in \Omega\times(0,T),\\ &u(x,t) = \lambda\eta(x,t), \qquad \text{for } (x,t) \in Q_{-\tau}.\end{aligned} \tag{3.11}$$

Now we are in a position to estimate the L^2-norm of u. Thus we need to choose a well posed φ. By the density of $\mathring{C}^\infty(Q_T)$ in E, for any given $t > 0$, we take $\varphi = u \cdot \chi_{[0,t]}(s)$ in the first equation of (3.11), where, $\chi_{[0,t]}$ is the characteristic function of $[0, t]$, then we obtain

$$\begin{aligned}&\frac{1}{2}\iint_{Q_t} \frac{\partial u^2}{\partial t} dxds + \lambda^{2-p} \iint_{Q_t} |\nabla u|^p dxds \\ =& \lambda \iint_{Q_t} (a|u|^{q_1-1}u + b|u_\tau|^{q_2-1}u_\tau)u(x,t)dxds,\end{aligned}$$

dropping the second terms of the left side, and integrating by parts yields

$$\begin{aligned}\int_\Omega u^2(x,t)dx &\le \int_\Omega \eta_0^2 dx + 2\lambda \iint_{Q_t} (a|u|^{q_1-1}u + b|u_\tau|^{q_2-1}u_\tau)u(x,t)dxds \\ &\le \int_\Omega \eta_0^2 dx + 2\lambda|a| \left(\iint_{Q_t} u^2 dxds\right)^{\frac{q_1+1}{2}} |Q_T|^{\frac{1-q_1}{2}} \\ &+ 2\lambda|b| \left(\iint_{Q_t} |u_\tau|^{2q_2} dxds\right)^{\frac{1}{2}} \left(\iint_{Q_t} u^2 dxds\right)^{\frac{1}{2}} \\ &\le \int_\Omega \eta_0^2 dx + 2\lambda|a| \left(\iint_{Q_t} u^2 dxds\right)^{\frac{q_1+1}{2}} |Q_T|^{\frac{1-q_1}{2}} \\ &+ 2\lambda|b| \left(\iint_{Q_t} |u_\tau|^2 dxds\right)^{\frac{q_2}{2}} \left(\iint_{Q_t} u^2 dxds\right)^{\frac{1}{2}} \cdot |Q_T|^{\frac{1-q_2}{2}},\end{aligned} \tag{3.12}$$

noticing that

$$\begin{aligned}\left(\iint_{Q_t} |u_\tau|^2 dxds\right)^{\frac{q_2}{2}} &\le \left(\iint_{Q_{-\tau}} |\eta|^2 dxds + \iint_{Q_t} u^2 dxds\right)^{\frac{q_2}{2}} \\ &\le \left(\iint_{Q_{-\tau}} |\eta|^2 dxds\right)^{\frac{q_2}{2}} + \left(\iint_{Q_t} u^2 dxds\right)^{\frac{q_2}{2}},\end{aligned}$$

substitute the above inequality into (3.12), yields

$$\begin{aligned}&\int_\Omega u^2(x,t)dx \le \int_\Omega \eta_0^2 dx + 2\lambda|Q_T|^{\frac{1-q_1}{2}}|a| \left(\iint_{Q_t} u^2 dxds\right)^{\frac{q_1+1}{2}} \\ &+ 2\lambda|b| \cdot |Q_T|^{\frac{1-q_2}{2}} \|\eta\|^{q_2}_{L^2(Q_{-\tau})} \left(\iint_{Q_t} u^2 dxds\right)^{\frac{1}{2}} + 2\lambda|b| \cdot |Q_T|^{\frac{1-q_2}{2}} \left(\iint_{Q_t} u^2 dxds\right)^{\frac{1+q_2}{2}}.\end{aligned} \tag{3.13}$$

For convenience, we denote

$$\begin{aligned}M_0 = \int_\Omega \eta_0^2 dx, \qquad M_1 = 2|Q_T|^{\frac{1-q_1}{2}}|a|, \\ M_2 = 2|b| \cdot |Q_T|^{\frac{1-q_2}{2}} \|\eta\|^{q_2}_{L^2(Q_{-\tau})}, \qquad M_3 = 2|b| \cdot |Q_T|^{\frac{1-q_2}{2}},\end{aligned}$$

obviously, M_0, M_1, M_2, M_3 are bounded constant. As is well known that for any $\alpha \geq 0$, $0 \leq r \leq 1$, we have

$$\alpha^r \leq 1 + \alpha.$$

Applying the inequality above to (3.13), yields

$$\begin{aligned}\int_\Omega u^2(x,t)dx &\leq M_0 + M_1(1 + \iint_{Q_t} u^2 dxds) + M_2(1 + \iint_{Q_t} u^2 dxds) + M_3(1 + \iint_{Q_t} u^2 dxds) \\ &= M_0 + M_1 + M_2 + M_3 + (M_1 + M_2 + M_3) \iint_{Q_t} u^2 dxds. \quad (3.14)\end{aligned}$$

Denote $\overline{M} = M_0 + M_1 + M_2 + M_3$, $\overline{M}' = M_1 + M_2 + M_3$, and apply Gronwall's inequality to above, yields

$$\int_\Omega u^2(x,t)dx \leq \overline{M}\exp(\overline{M}'t).$$

Therefore, we have

$$\iint_{Q_T} u^2(x,t)dxdt \leq \overline{M}/\overline{M}' \cdot (e^{\overline{M}'T} - 1).$$

Furthermore, we have

$$\iint_{Q_T} u^2(x,t)dxdt \leq \iint_{Q_{-\tau}} \eta^2 dxdt + \overline{M}/\overline{M}' \cdot (e^{\overline{M}'T} - 1).$$

The above inequality implies that the set

$$\{u \in X; u = \lambda F(\Phi(u)) \text{ for some } 0 \leq \lambda \leq 1\}$$

is bounded uniformly. According to Schaefer's Fixed Point Theorem, $F(\Phi(\cdot))$ has fixed point. That is there exists a function $u \in X$, such that $u = F(\Phi(u))$. From the definition of $F(\Phi(\cdot))$, and combining with equation, we further derive $u \in E$ is a generalized solution of the initial and boundary value problem (1.1)-(1.3). From the proof above, we see that $T > 0$ is arbitrary, which meas that the solution exist globally.

Next, we show the uniqueness. Let u, v with $u \not\equiv v$ be two solutions of the problem (1.1)–(1.3), then we see that

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\int_\Omega (u-v)^2 dx + \int_\Omega (|\nabla u|^{p-2}\nabla u - |\nabla v|^{p-2}\nabla v)\nabla(u-v)dx \\ = &\int_\Omega \left(|u|^{q_1-1}u - |v|^{q_1-1}v\right)(u-v)dx + \int_\Omega \left(|u_\tau|^{q_2-1}u_\tau - |v_\tau|^{q_2-1}v_\tau\right)(u-v)dx\end{aligned}$$

for any $t \leq \tau$, we see that $u_\tau(x,t) = v_\tau(x,t)$, and note that $q_1 \leq 1$, then for any $t \leq \tau$, we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\int_\Omega (u-v)^2 dx \leq \int_\Omega \left(|u|^{q_1-1}u - |v|^{q_1-1}v\right)(u-v)dx \\ \leq &\int_\Omega (u-v)^{q_1+1}dx \leq \left(\int_\Omega (u-v)^2 dx\right)^{\frac{q_1+1}{2}},\end{aligned}$$

which means that

$$\int_\Omega (u(x,t)-v(x,t))^2 dx \equiv 0, \text{ for any } t \leq \tau.$$

By an iterative process, it is easy to see that

$$\sup_{t\geq 0}\left\{\int_\Omega (u(x,t)-v(x,t))^2 dx\right\} \equiv 0.$$

The uniqueness is proved.

While if $a = 0$, $b \neq 0$, for any $q_2 \geq 0$, the solution will exists globally for any initial data. In fact, by using the method in [10], one can first study this problem on $[0, \tau]$, then study it on $[\tau, 2\tau]$, $\cdots$, $[k\tau, (k+1)\tau]$, By an iterative process, for any $T > 0$, the solution exists on $[0, T]$. □

Remark 3.1. In fact, if $a > 0$, $b = 0$, the solution may blow up for some initial data when $q_1 > \max\{1, p-1\}$, see for example [7, 11] and the reference therein.

4 Conclusion

In this paper, we establish the existence and uniqueness of global solution for a time delayed p-Laplace equation. In fact, the delay term will not affect the global existence of solutions, that is blow-up is impossible for such kinds of delayed equation if $a = 0$. But for the nonlocal time delay, blow-up will happen[10].

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Competing Interests

Author has declared that no competing interests exist.

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